

ESSAYS ON FRICTIONS AND INEFFICIENCY IN HOUSING AND REGIONAL LABOUR MARKETS

by

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Abstract

These essays examine the efficiency and concomitant policy implications of various frictions (financial, trading, labor market or policy constraints) in housing and regional labor markets. These frictions generate externalities that are either pecuniary (as in the first two chapters) or demand based (the third chapter) that in turn induce inefficiencies along different margins. The essays thereby apply a normative approach that focuses on characterizing inefficiencies and deriving associated corrective policies.

The first chapter describes the inefficiencies associated with mortgage defaults in a standard equilibrium housing model that is calibrated to the U.S. housing market. The inefficiencies are evaluated in a simulated downturn that captures the peak foreclosure spike and house price decline in the Great Recession. The deadweight cost inefficiency associated with realized lender losses from foreclosure are found to dominate pecuniary externalities, which are insignificant. Debt renegotiation mitigates lender losses following default but might be inefficiently low when transaction costs are incurred prior to the renegotiation process.

The second chapter studies the constrained inefficiency of house sale choices in a heterogeneous agent frictional model of the housing market. Trading frictions are modeled using a broker-intermediated directed search framework. Pecuniary externalities arise due to imperfect risk-sharing between agents and induce inefficiently high sales if house sellers are more constrained in the aggregate than buyers. Under the same condition, a novel finding is that private sellers also list prices which are

ABSTRACT

lower than the efficient list price.

The third chapter studies regional labour mobility in an economy where adverse rural labour demand shocks lead to binding downward nominal wage rigidity and the monetary policy response is limited. It shows the constrained inefficiency of individual regional labour mobility choices due to an aggregate demand externality, thereby reaffirming and extending prior insights from [Farhi and Werning \[2014\]](#). The output multiplier of a policy encouraging labour mobility following adverse shocks is related to fiscal multipliers associated with regional transfer policies, with the former being significant when demand linkages are stronger and home bias for regional goods is weaker.

Keywords: Default; Housing; Pecuniary externalities; Deadweight cost; Great Recession; Directed search; Downward nominal wage rigidity; Rural-urban migration; Demand externality

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This dissertation is dedicated to my family: my parents, sister and brother-in-law, for their unwavering love and support.

Dedication

This thesis is dedicated to my family.

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Chapter 1

Inefficient mortgage defaults: theory and application to the housing downturn

1.1 Introduction

A recent literature has characterized inefficiencies and optimal corrective policy in macroeconomic models of fire sales and pecuniary externalities (e.g. [Dávila and Korinek \[2018\]](#)). This literature predominantly employs stylized (three-period) models in representative agent settings and is *normative* in nature: it typically studies the *constrained efficiency* of individual choices in frictional environments, i.e. whether a policymaker maximizing social welfare subject to the same market incompleteness frictions as individuals would make different choices. These inefficiencies in turn motivate remedial policy interventions.

A distinct recent literature has studied the causes of the 2006-2011 housing bust in the U.S. and the impact of policy in mitigating the significant foreclosure spike and house price decline (examples include [Ganong and Noel \[2020a\]](#), [Kaplan et al. \[2020\]](#)). However, these articles do not investigate the inefficiencies that have often

been cited as motivation for foreclosure reduction policies during the bust (see, for e.g., the discussion in [Agarwal et al. \[2017\]](#), pp. 660-661).

The objective of this paper is to employ the aforementioned normative framework to study foreclosure reduction policies in the Great Recession. Specifically, I characterize and evaluate the inefficiencies and externalities associated with default choice using a standard incomplete markets housing model with long-term defaultable or saleable mortgages. These inefficient individual default (and foreclosure) choices motivate policy aimed at reducing foreclosures. This is the first paper to describe the inefficiencies associated with debt default decisions and then quantify these inefficiencies in an applied setting, namely the mortgage market in the Great Recession.

There are two principal inefficiencies associated with mortgage default choice:

1. There is an inefficiency associated with lender loss from foreclosure (*foreclosure deadweight cost*) following default¹. The foreclosure loss is the amount a lender can recover from a foreclosure sale less the loan balance owed to them. The Social planner (policymaker) accounts for realized lender losses from foreclosure in the social welfare criterion. An owner choosing default would compare his private values from defaulting versus sale or repayment, and would not account for the foreclosure loss faced by lenders. Clearly, lender losses would induce the planner to lower foreclosure intensity. The foreclosure loss faced by lenders can be partially mitigated if mortgage renegotiation is possible. Hence, one can interpret this as a **renegotiation inefficiency** arising in part due to frictions associated with the lack of renegotiation.
2. Default (or sale) by an agent affects house prices by increasing housing supply. The resulting price movements affect the net worth of asset holders, and

¹These are sometimes referred to as the cost of distress of default. These deadweight costs share some features of externalities, but are not strictly externalities as they are borne by a party to the loan transaction.

collateral constraints (loan-to-value constraints) faced by home buyers. There are **pecuniary externalities** associated with mortgage default, due to market incompleteness (*distributive externalities*) and collateral constraints (*collateral externalities*). Market incompleteness and imperfect risk sharing imply that price movements that affect the net worth of different agents do not have a net zero effect. Price movements also affect collateral value and could lead to binding constraints for some homebuyers². If default pecuniary externalities are negative, they would induce the planner to choose a lower default and foreclosure rate.

The efficient default choice is the solution to a constrained optimization problem where the social planner chooses default on behalf of owners in order to maximize a utilitarian social welfare function. The planner is subject to market incompleteness and financial frictions, the decentralized choices of individual agents and equilibrium pricing relationships. The planner's intervention is unanticipated by agents, i.e. I conduct an *ex post* analysis.

The model is calibrated to match certain pre-crisis data moments. The calibrated model is then subjected to a combination of unanticipated one-time credit tightening, buyer transaction cost and negative income shocks to generate a house price decline ($\sim 20\%$) and foreclosure rate spike (an increase of $40 - 50\%$) approximating that witnessed during the worst phase of the housing downturn in the Great Recession (see Figure 1.1).

I consider the constrained efficient default rate and corrective policy during the *simulated* downturn. Hence, I evaluate inefficiencies in the worst phase of the housing market bust. I also compute remedial *ex post* policy, expressed in terms of either a debt reduction or a cash-in-hand subsidy. Finally, I use a parsimonious specification of debt renegotiation following default to consider whether renegotiation significantly lowers

²The classification of pecuniary externalities is due to [Dávila and Korinek \[2018\]](#). I describe these externalities in more detail in section 1.3.5.

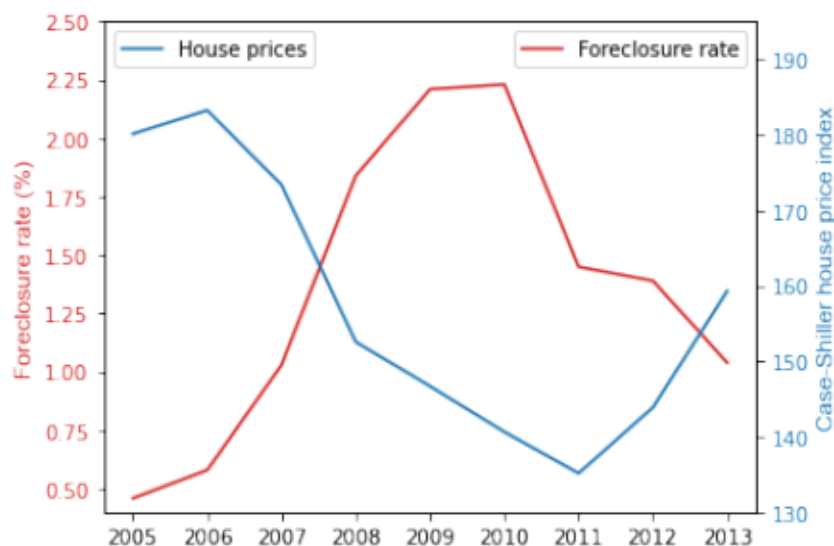


Figure 1.1: House prices and foreclosures in the U.S., 2005-2017.

Notes: During the crisis, house prices (right-axis) fell by almost 20% and the foreclosure rate (left-axis) rose by over 40% (house price series indexed to 100 in 2000).

socially inefficient foreclosures (i.e. when there are unrealized gains from mortgage renegotiation).

The results can be summarized as follows:

1. Lender loss from foreclosure (the foreclosure deadweight cost) is the dominant inefficiency in the downturn. Incorporating lender losses from foreclosure in the planner's default choice leads to a constrained efficient default rate of 2.08% relative to the decentralized equilibrium default rate of 2.3%.
2. Renegotiation mitigates lender losses, but might be lower than the socially efficient level if renegotiation entails incurring *ex ante* transaction costs.
3. The net pecuniary externality associated with default is negative but tiny. I find that the efficient default rate accounting solely for pecuniary externalities is quantitatively equivalent to the decentralized equilibrium default rate.
4. Inefficiency correction (through corrective debt reduction) accounts for a small

share (approximately 10%) of the average debt reduction under the Home Affordable Modification Program's (HAMP) Principal Reduction Alternative.

The first result arises due to the large price decline in the downturn that reduced the recovery amounts for lenders from foreclosure. It accords well with the estimate from [US Department of Housing and Urban Development \[2010\]](#) that lender losses from foreclosure are sizable and far exceed their estimate of consumer losses from foreclosure associated with moving costs, legal and administrative fees. The final result indicates that, *if* correcting inefficiencies was the predominant reason for foreclosure reduction policies in the downturn, then foreclosure inefficiencies were smaller than policymakers presumed.

An interesting finding is that pecuniary externalities are extremely small in this environment. There are two main reasons for the small magnitude of the pecuniary externalities in the simulated crisis. *First*, as prices fall and home equity becomes negative (i.e. when the loan balance exceeds the house value), owners choose to default rather than sell. Defaults lead to foreclosures, and losses due to lower prices are borne by unconstrained, risk-neutral lenders rather than constrained sellers. This lowers the magnitude of the overall distributive externality. *Second*, collateral externalities are small due to a combination of loan pricing that endogenizes default risk and a precautionary motive that induces agents to save and not leverage up significantly.

An objective of this paper is to assess the quantitative importance of default inefficiencies in the design of *ex post* policy in this environment. The focus on *ex post* (crisis mitigation) policy is mainly due to the difficulty in deriving optimal corrective *ex ante* (macroprudential) policy in a model with long-term debt.

1.1.1 Related literature

The literature on mortgage modifications during the Great Recession is largely empirical, focusing on the reasons for low private modification rates (e.g. Agarwal et al. [2011], Piskorski et al. [2010]) and analyzing policies like the Home affordable modification program (HAMP) that promoted modifications (e.g. Agarwal et al. [2017], Ganong and Noel [2020a]). My treatment of renegotiation is based on the analysis of renegotiation of one-period defaultable sovereign debt in Yue [2010]. Inefficiency due to hold-up problems associated with *ex ante* transaction costs are discussed by Anderlini and Felli [2001, 2006].

Pecuniary externalities and constrained inefficiency arising therefrom have been discussed extensively in various environments (see Dávila and Korinek [2018] and the references therein). These papers typically consider environments with limited agent heterogeneity and short-term non-defaultable debt. Hence they have not hitherto analysed potential inefficiencies associated with mortgage default and foreclosure decisions in the housing market. Further, the assumption of non-defaultable debt implies that the renegotiation inefficiency has not been considered by these papers. Though the emphasis in this literature has been on intervention *ex ante*, there is a smaller literature that evaluates the use and implications of *ex post* intervention (e.g. Jeanne and Korinek [2020], Bianchi [2016]). Pecuniary externalities in a standard incomplete markets model were first investigated by Davila et al. [2012].

The quantitative model used below is the standard incomplete markets-heterogeneous agent equilibrium model that is adapted to the housing market. Other examples include Chatterjee and Eyigungor [2015], Garriga and Hedlund [2020]. Kiyotaki et al. [2011] consider distributional effects of house price changes in a life-cycle incomplete markets model with multiple house sizes, convertibility between rental and owner-occupied housing but without default. However, they do not consider pecuniary

externalities and corrective policy.

The model below features a negative impact of foreclosures and sales on house prices through a supply channel. Empirical evidence on the spillover/externality effects of foreclosures on prices is mixed, with articles like [Campbell et al. \[2011\]](#) finding small localized effects while [Mian et al. \[2015\]](#), [Guren and McQuade \[2020\]](#) demonstrate that foreclosures can significantly lower house prices, empirically and quantitatively. However, this literature does not model pecuniary externalities and consider inefficiency arising therefrom.

A distinct approach is to design optimal mortgage contracts *ex ante* (e.g. [Piskorski and Tchisty \[2010\]](#), [Guren et al. \[2018\]](#), [Campbell et al. \[2020\]](#)). The *ex ante* normative analysis described above in contrast treats the debt structure as given and derives efficient macroprudential policy.

Outline of paper

Section [1.2](#) describes the quantitative model. Section [1.3](#) discusses the inefficiencies associated with default choice, and Section [1.4](#) discusses calibration of the model and the shocks applied to the stochastic steady state. Section [1.5](#) then describes the simulated downturn and the constrained efficient default choice, and Section [1.6](#) discusses corrective policy following the shock. Section [1.7](#) discusses some extensions to the baseline model, and section [1.8](#) concludes. Supplementary material in the appendices are described at the beginning of Appendix [1.9.1](#).

1.2 Model

The model employed for the quantitative analysis is a standard infinite horizon incomplete markets model with housing, with heterogeneity within agent types *ex post* arising due to saving and borrowing choices of agents facing uninsurable idiosyncratic

risk. There are three types of agents: homeowners, renters and lenders. Owners of housing take out mortgages in order to purchase a house, which they may default upon or terminate through sale. Default leads to foreclosure and affects an owner's credit history for a stochastic period of time, to be discussed below. At any point in time, an agent who is not a lender in this economy can be characterized by his income, asset and debt positions (for owners), but also by whether he is an owner, renter or a renter with a bad credit history (with a default flag).

1.2.1 Environment

Time is discrete, continues forever and is indexed by $t = 0, 1, 2, \dots$. There is a continuum of agents who receive an endowment y drawn independently according to a Markov process with values in finite set Y . The probability that an endowment transitions from current level of y to y' is given by the transition matrix $\Pi(y'|y)$. Agents have period utility functions $u(c, \chi h)$, hence they receive benefits from consumption and housing services if they are owners (the housing preference parameter $\chi > 1$ for owners). For simplicity, it is assumed that rental housing yields housing services one-for-one with house size, i.e. $\chi = 1$. Utility is separable between consumption and housing services. All agents discount the future using discount factor β .

As described below, owners also face idiosyncratic depreciation shocks δ_h , which are binary ($\{\delta_h^l, \delta_h^h\}$ with probabilities $\{1 - \xi, \xi\}$ respectively). This assumption is needed in order to obtain default in steady state³, and is common to other articles in the literature (e.g. [Chatterjee and Eyigungor \[2015\]](#), [Arslan et al. \[2015\]](#), [Garriga and Hedlund \[2020\]](#)).

³Models featuring aggregate shocks (e.g. [Kaplan et al. \[2020\]](#)) assume a utility cost associated with defaulting in order to calibrate the equilibrium foreclosure rate.

Mortgages

Mortgages are long-term debt contracts that home buyers enter into with lenders. In an infinite horizon setting, the mortgage is treated as a perpetuity with geometrically decaying (at rate δ_m) loan balance (as in e.g. [Chatterjee and Eyigungor \[2015\]](#)). Let the risk-free rate be denoted by r , with $R = (1+r)$, and the loan balance at origination be b . Let the initial payment (installment) be denoted by m . m is chosen such that the sequence of payments $\{m, \delta_m m, \delta_m^2 m, \dots\}$ solves the following equation:

$$b = \frac{m}{(1+r)} + \frac{\delta_m m}{(1+r)^2} + \frac{\delta_m^2 m}{(1+r)^3} + \dots \quad (1.1)$$

Then, the payment is $m = b(1+r-\delta_m)$. With this sequence of payments, note that the loan balance next period is $b' = b(1+r) - m = \delta_m b$. In this setup, the payment sequence is determined at origination, although the periodic installments are not equal. As shall be discussed below, the mortgage contract is defaultable/saleable and is collateralized by the value of the house.

Housing market

The treatment of the housing market is straightforward, in order to quantitatively evaluate the efficient default level while maintaining analytical tractability.

Agents can either rent or own a house. Housing (owner-occupied and rental) is of a single size ($h = 1$), and agents can only own one house at a time. There is a fixed housing stock normalized to 1, and no construction. The owner-occupied and rental sectors of the housing market are segmented, so there is no convertibility between owner-occupied and rental housing space, which would otherwise pin down a relationship between house prices and rents (see e.g. [Kaplan et al. \[2020\]](#)). This also implies that the homeownership rate is constant.

Rents are fixed so as to maintain a fixed rent-income ratio (discussed in section 1.4 below), and are assumed to be earned by absentee (un-modeled) landlords. This assumption is made primarily for tractability, otherwise the equilibrium rent would need to be determined and the pecuniary externality operating through rents would also be operative.

Under these assumptions, demand for owner-occupied housing by renters must be met by supply from existing owners, in the form of either foreclosures or sales. The house price adjusts to clear the owner-occupied housing space, and the rental market absorbs rental demand at the fixed rent. Hence, the price-rent ratio moves along with house prices. The equilibrium condition in the (owner-occupied) housing market is stated below.

1.2.2 Value functions and choices

The value functions and choices associated with different types of agents can be summarized as follows:

- Current owners can *continue* with their mortgage payments, *sell* their home or *default* on the mortgage
- Renters can *buy* a house and become an owner, or continue to *rent*
- Agents with a default flag (bad credit history) rent and are excluded from the owner-occupied housing market (referred to as *autarky* below)

In the notation to follow, the short term constraint that precludes borrowing, i.e. $a'(s) \geq 0$, is represented by the constrained choice set $a' \in A \subseteq \mathbb{R}_+$. I refer to the saving and loan choice sets by A and B respectively, and the set of depreciation shock values by Δ_h below.

Current owner

As mentioned above, a current owner each period chooses whether to continue repaying, sell his house or default. Upon default or sale, the agent loses possession of his house immediately and rents in that period. Default ($\delta = 1$) is distinguished from sale ($\sigma = 1$) as the defaulter is excluded from the housing market for a stochastic period of time. The state variables for an owner in each period include his endowment y , asset position a , the beginning-of-period loan balance b and the depreciation shock faced by owners, δ_h . Let the vector of state variables be $s^o = (y, a, b, \delta_h) \in S^o = Y \times A \times B \times \Delta_h$. The expectations operator \mathbb{E} is defined over (y', δ'_h) given s^o using transition probabilities Γ and high depreciation shock probability ξ .

I represent the value of **continuing** with a mortgage contract by $V^c(s_t^o)$. Then, the Bellman equation for continuing is:

$$V^c(s_t^o) = \max_{\{c, a' \in A\}} u(c, \chi h) + \beta \mathbb{E} \max \left\{ V^c(s_{t+1}^o), V^d(s_{t+1}^o), V^s(s_{t+1}^o) \right\} \quad (1.2)$$

subject to

$$c + a' = y + aR - m - \delta_h p h$$

If the owner chooses to **sell** his house ($\sigma(s_t^o) = 1$), he keeps the proceeds from selling his house net of transaction cost ($\kappa_h p h$) after paying off his remaining loan balance (b). The depreciation shock is interpreted as a random maintenance cost borne by continuing owners and sellers. The Bellman equation for sale is:

$$V^s(s_t^o) = \max_{\{c, a' \in A\}} u(c, h) + \beta \mathbb{E}_{y'} V^r(y', a') \quad (1.3)$$

subject to

$$c + a' = y + aR + ph(1 - \delta_h - \kappa_h) - b$$

If the owner chooses to **default** on his mortgage ($\delta(s_t^o) = 1$), then he is automatically foreclosed. He receives a default flag and is temporarily excluded from the housing market. His value function is:

$$V^d(s_t^o) = \max_{\{c, a' \in A\}} u(c, h) + \beta \mathbb{E}_{y'} V^{aut}(y', a') \quad (1.4)$$

subject to

$$c = y + aR + \max\{\kappa ph - b, 0\} - a'$$

The owner value function for each state vector is defined as the upper envelope of the value functions associated with continuation, sale and default defined at the same state vector:

$$V^o(s_t^o) = \max \left\{ V^c(s_t^o), V^d(s_t^o), V^s(s_t^o) \right\} \quad (1.5)$$

Renter

The renter can either purchase a home through a mortgage ($\omega = 1$), in which case he becomes an owner in the following period, or choose to remain a renter ($\omega = 0$), which is referred to as tenancy. The relevant state variables for a renter are his income y and asset level a , so a renter's state vector $s^r = (y, a) \in S^r = Y \times A$.

A renter who chooses not to purchase a house remains a **tenant**, paying rent ρ , and has the value function:

$$V^t(s^r) = \max_{\{c, a' \in A\}} u(c, h) + \beta \mathbb{E}_{y'} V^r(y', a') \quad (1.6)$$

subject to

$$c + a' + \rho = y + aR$$

A renter who chooses to **buy** a house will do so by purchasing a mortgage. Given house value ph and transaction cost $\kappa_b ph$, his initial loan amount would be qb' . The buyer is also subject to a loan-to-value (LTV) constraint with downpayment fraction ι given by:

$$q(y, a', b')b' \leq (1 - \iota)ph \quad (1.7)$$

It is assumed that a buyer enjoys homeownership utility premium in the period of purchase, i.e. he gets immediate possession of the house. Therefore, his value function would be:

$$V^b(s^r) = \max_{\{c, a', b'\}} u(c, \chi h) + \beta \mathbb{E}_{y', \delta'_h} V^o(y', a', b', \delta'_h) \quad (1.8)$$

subject to the LTV constraint (1.7) and the budget constraint,

$$c + a' = qb' - ph(1 + \kappa_b) + aR + y$$

The renter value function for each state vector is defined as the upper envelope of the tenant's and homebuyer's value functions defined at the same state vector:

$$V^r(s^r) = \max \{V^t(s^r), V^b(s^r)\} \quad (1.9)$$

Agents with a default flag

Agents who have defaulted receive a default flag (i.e. have a bad credit record) and are excluded from the owner-occupied housing market, hence they rent housing. The default flag is assumed to be removed stochastically (based on a Poisson shock) with

probability θ (hence a defaulter is excluded for $\frac{1}{\theta}$ periods). Default is therefore *costly* in terms of being excluded from the owner-occupied housing market for a random length of time. The state variables for an agent with a default flag are income y and asset level a , hence the state vector is $s^{aut} = (y, a) \in S^{aut} = Y \times A$.

The autarky value function is:

$$V^{aut}(y, a) = \max_{\{c, a' \in A\}} u(c, h) + \beta \mathbb{E}_{y'} \left\{ (1 - \theta) * V^{aut}(y', a') + \theta * V^r(y', a') \right\} \quad (1.10)$$

subject to the borrowing constraint:

$$c = y + aR - \rho - a'$$

1.2.3 Lender behavior

Loans are priced by competitive risk-neutral lenders on an individual basis, following the literature initiated by Chatterjee et al. [2007]. If q is the price of the loan in the current period and B is the loan size, loans are priced for an individual with current income y and saving and loan balance for next period given by $\{a', b'\}$, i.e. owner state $s'^o = (y', a', b', \delta'_h)$, such that lenders break even in expectation:

$$q(y, a', b')b' = \frac{\mathbb{E}_{y', \delta'_h} \left[\delta(s'^o) * \min \{ \kappa p' h, b' \} + \sigma(s'^o) * b' + \left((1 - \delta(s'^o)) (1 - \sigma(s'^o)) \right) * (m(b') + q'(y', a'', b'')b'') \right]}{R} \quad (1.11)$$

This is a recursive relationship that describes how the loan price evolves over time. In the event of default ($\delta = 1$), the house is foreclosed upon and the lender recovers a fraction κ of the proceeds⁴. In the event of sale ($\sigma = 1$), the lender recovers the loan balance. If the owner continues with the mortgage repayment plan, the lender receives the current period payment and the loan balance evolves according to the geometric decay formula.

⁴In common with much of the literature, I assume that mortgages are non-recourse loans.

1.2.4 Stationary equilibrium

A stationary equilibrium consists of value functions $\{V^c, V^s, V^d, V^{aut}, V^b, V^t, V^r, V^o\}$, decision rules $\{a'^{aut}, a'^r, a'^o, \omega, \sigma, \delta, B'\}$, distributions of owners, renters and agents with a default flag $\{\mu^O, \mu^R, \mu^{aut}\}$, and house prices p and loan prices q that satisfy:

1. Owners and renters make their choices as described in section 1.2.2 given house price p and rent ρ
2. Loan prices satisfy equation (1.11)
3. House prices equate demand and supply in the owner-occupied housing market:

$$\int_{s^r \in S^r} \omega(s^r; p) d\mu^R(s^r) = \int_{s^o \in S^o} \sigma(s^o; p) d\mu^O(s^o) + \int_{s^o \in S^o} \delta(s^o; p) d\mu^O(s^o)$$

4. The distributions $\{\mu^j\}_{j=R,O,aut}$ are consistent with individual sale (σ), default (δ) and purchase choices (ω). The updating of the distributions is described in appendix 1.9.3.

1.3 Inefficiencies associated with default choice

The notion of efficiency considered here is *constrained efficiency*: a social planner or policymaker chooses default for owners subject to the market incompleteness frictions, collateral constraints and taking as given the other individual decentralized choices (e.g. saving, borrowing, ownership) and the equilibrium price relationships. Based on the approach in Davila et al. [2012], I consider whether the planner could improve on the decentralized outcome through a different choice of default, without correcting market incompleteness. Thus, the planner only chooses whether an owner defaults or not, taking as given all other policy functions from the decentralized problem and internalizing the impact of default choice on equilibrium house prices. The planner's choice is also subject to the equations of motion for the distributions, described in appendix 1.9.3. The analysis here is *ex post* in nature, hence the planner takes loan prices, debt choices and the distributions of agents entering into the period as given when choosing whether to default.

A potential complication is the forward-looking nature of loan pricing in equation (1.11). If default choices by a planner are anticipated, this would change lender and borrower behaviour in prior periods. In this paper, I assume that the planner's intervention is unanticipated and agents do not account for a possible intervention that would reduce default risk in the future.

1.3.1 Decentralized default choice

First, consider the default choice of an individual agent, which is referred to as the decentralized default choice. The default choice is a binary one. In what follows, some of the arguments of the value or policy functions are suppressed for notational convenience.

The value functions in equations (1.5) and (1.9) can be rewritten in terms of the optimal choices of ownership, $\omega(y, a)$, default, $\delta(y, a, b, \delta_h)$, or sale, $\sigma(y, a, b, \delta_h)$:

$$V^o(s^o) = (1 - \delta(s^o)) \left\{ (1 - \sigma(s^o)) * V^c(s^o) + \sigma(s^o) * V^s(s^o) \right\} + \delta(s^o) * V^d(s^o) \quad (1.12)$$

$$V^r(y, a) = \omega(y, a) * V^b(y, a) + (1 - \omega(y, a)) * V^t(y, a) \quad (1.13)$$

An owner with state $s^o = (y, a, b, \delta_h)$ chooses default ($\delta = 1$) if:

$$V^d(y, a, b) > \max \left\{ V^s(y, a, b, \delta_h), V^c(y, a, b, \delta_h) \right\} \quad (1.14)$$

1.3.2 Constrained social planner's problem

As mentioned above, the planner chooses default ($\delta(s^o)$) on behalf of owners in order to maximize social welfare, subject to the other decentralized choices, financial constraints and equations of motion for the distributions.

The Social Welfare Function (SWF) assumed here is utilitarian: it sums up all agents' utilities and also includes lenders' receipts in the event of defaults (leading to foreclosures). This follows Davila et al. [2012], and can be motivated by the assumption in standard incomplete markets models that agents are *ex ante* identical.

Lenders are included despite the fact that they account for potential foreclosure losses and make zero profits *ex ante*. Their foreclosure losses depend on the realization of the house price, which affects their recovery amount. The planner is assumed to care about realized lender foreclosure losses from a welfare perspective. This is realistic, and reflects broader concerns of policymakers about the health of the financial sector considering its role in amplifying shocks (e.g. [Gertler and Kiyotaki \[2010\]](#)). Further, the deadweight loss of foreclosure only affects welfare if lender losses are incorporated in the SWF. Finally, these lender losses enter additively in the SWF as they are valued by risk-neutral lenders.

The masses of owners, renters and agents with default flags in the economy are denoted by $d\mu^O(y, a, b, \delta_h)$, $d\mu^R(y, a)$ and $d\mu^{aut}(y, a)$, and are determined as part of the equilibrium.

Extending the recursive definition of [Davila et al. \[2012\]](#) to the setting here, the planner's problem becomes:

$$\begin{aligned}
 \Omega(\mu^O, \mu^R, \mu^{aut}) = & \max_{\{\delta(y, a, b, \delta_h)\}} \sum_{y \in Y} \int_{a \in A} u(c, h) * d\mu^R(y, a) + \sum_{y \in Y} \int_{a \in A} u(c, h) * d\mu^{aut}(y, a) \\
 & + \sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} \left(\delta(y, a, b, \delta_h) \min\{\kappa p h, b\} + (1 - \delta(y, a, b, \delta_h)) b \right) * d\mu^O(y, a, b, \delta_h) \\
 & + \sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} u(c, \chi h) * d\mu^O(y, a, b, \delta_h) \\
 & + \beta \mathbb{E} \Omega(\mu'^O, \mu'^R, \mu'^{aut})
 \end{aligned} \tag{1.15}$$

subject to the decentralized choices $\{\omega, \sigma, a', b'\}$, equation (1.11), the owner-occupied housing market clearing condition and the updating operator T for distributions:

$$(\mu'^O, \mu'^R, \mu'^{aut}) = T(\mu^O, \mu^R, \mu^{aut}; Q(y, a, b, \delta_h, \mu'^O, \mu'^R, \mu'^{aut}, \cdot))$$

where $Q(\cdot, \cdot)$ is the equation of motion for the distributions that is described in appendix 1.9.3.

Hence, the planner chooses whether to default for each owner in order to maximize

the SWF objective subject to the financial frictions, decentralized choices and equations of motion for the distributions. Clearly, the planner's default choice affects the intensity of sale or repayment. I assume that if the planner chooses not to default on behalf of an owner, then the choice between sale or repayment is made by the owner. Hence, the planner only intervenes on the margin of whether to default or not.

1.3.3 Planner's default choice

It is easier to compare the planner's default choice to the decentralized default choice using an alternative expression for the SWF objective based on the definitions of choice-specific value functions (described in appendix 1.9.4) rather than equation (1.15). It is a simple exercise to show the equivalence of these two representations, using the expressions for the transition of distributions provided in appendix 1.9.3. Hence, the equations of motion for the distributions are embedded into this version of the problem.

Proposition 1: The planner chooses default ($\delta(y, a, b, \delta_h) = 1$) for an owner if:

$$V^d(y, a, b) - \max \left\{ V^s(y, a, b, \delta_h), V^c(y, a, b, \delta_h) \right\} + \underbrace{\min\{\kappa ph, b\} - b}_{\text{Lender loss from foreclosure}} + \underbrace{\text{PE}}_{\text{pecuniary externality}} > 0 \quad (1.16)$$

Proof: In appendix 1.9.6.

The wedge between decentralized and planner's default payoffs relative to choosing sale or repayment in equation (1.16) has three components. The first term is the expression for decentralized default choice, equation (1.14), the second term is the loss lenders face when default is chosen, and the third term is the pecuniary externality, i.e. the uninternalized effect of defaults on other agents in the economy operating through house prices. The complete expression for the pecuniary externality (PE) is in appendix 1.9.6.1.

Hence, the wedge between planner and decentralized default choice can be written as:

$$\text{Wedge}(b) = \underbrace{\min\{\kappa ph, b\} - b}_{\text{Lender loss from foreclosure}} + \text{PE}$$

If the wedge term is *negative*, then it would provide an additional force against choosing default.

The total pecuniary externality (PE) is given by the sum of the distributive and collateral externality terms:

$$PE = \text{Distributive externality} + \text{Collateral externality} \quad (1.17)$$

The distributive externality term is considerably involved, but can generally be expressed as follows. Let the net worth of an agent with state s^j be denoted by $NW(s^j)$. Then, a concise representation of the distributive externality is given by:

$$\text{Distributive externality} = \sum_{\{s^j \in S^j\}_{j=\{o,r,a,b\}}} \left[\text{Valuation of } \Delta NW(s^j) * \frac{\Delta NW^j}{\Delta p} * d\mu^j(s^j) \right] * \frac{\Delta p}{\Delta H} \quad (1.18)$$

The complete expression for the distributive externality can be found in appendix 1.9.6.2. To illustrate, consider the distributive externality terms from appendix 1.9.6.2 for lenders and buyers. The valuation of net worth (or liquidity) changes for lenders is 1, while for buyers it is the marginal utility of consumption. The second term is the variation in net worth with prices. For lenders, this is the fraction κ recovered from a foreclosure sale, while for buyers it is $(1 + \kappa_b)$ ⁵.

The collateral externality, deriving from the loan-to-value constraint faced by borrowers (equation (1.7)) is:

$$\text{Collateral externality} = \left[\sum_{y \in Y} \int_{a \in A} \eta^c(y, a) * (1 - \iota) * d\mu^R(y, a) \right] * \frac{\Delta p}{\Delta H} \quad (1.19)$$

Here, $\eta^c(y, a)$ is the Lagrange multiplier on the LTV constraint of a buyer with state (y, a) which equals zero if the buyer is not constrained, and ι is the downpayment fraction.

The price impact of a termination by an owner is denoted by $\frac{\Delta p}{\Delta H} \leq 0$, and will be discussed further in section 1.3.5. Hence, the PE term is negative (and thereby

⁵The externality expressions and results derived therefrom are of course dependent on the model specification, hence my choice of a standard equilibrium housing model with default.

opposes default) if the sum of the terms in square brackets in equations (1.18 – 1.19) is positive.

Finally, note that the PE term does not depend on an individual's state. This is because it aggregates over the marginal impact on agents' net worth, and the price impact does not vary across individuals.

1.3.4 Lender loss from foreclosure and the lack of renegotiation

An important component of the expression in equation (1.16), and of the wedge between the planner's and decentralized default choices, is the lender loss from foreclosure: $\min\{\kappa ph, b\} - b \leq 0$. This is an *inefficiency*/deadweight cost associated with default choice by an owner. It arises because an owner only compares his private values of default versus its alternatives without incorporating the loss to other agents in the economy. Although lenders incorporate the possibility of loss from foreclosure *ex ante* in their loan pricing choice, the decision to default rather than sell or repay is taken by owners independently of lenders *ex post*.

These *ex post* foreclosure losses (deadweight costs or cost of distress of default for lenders) motivate policy intervention. Clearly, these deadweight costs of foreclosure favour fewer defaults and foreclosures. The fact that foreclosure losses are also lower when prices are higher factors into the distributive externality expression for lenders in appendix 1.9.6.2.

Foreclosure losses are indicative of the deadweight losses associated with foreclosure. This is evident from the presence of the foreclosure recovery fraction κ in the foreclosure loss expression. As the deadweight loss declines (i.e. κ rises), so does the lender-loss based inefficiency associated with default choice.

It would seem natural that if there were gains from renegotiation of debt contracts, i.e. if

$$V^d(y, a, b) - V^c(y, a, b, \delta_h) + \kappa ph - b < 0$$

then lenders and owners would choose to do so. Note that this expression comprises the first two terms of the planner's default choice from equation (1.16) if $V^c(y, a, b, \delta_h) > V^s(y, a, b, \delta_h)$. In the absence of pecuniary externalities, the planner would wish to

avoid default if the surplus from doing so is negative, i.e. if default (and foreclosure) is socially inefficient. Of course, if there were no gains from renegotiation, then the planner's default choice would not differ from that of the owner. Therefore, the presence of the wedge when the default choice is discrete does not necessarily imply that the private default choice is inefficient.

Efficiency of costless renegotiation choices

Suppose that the surplus from *renegotiation* is positive and renegotiation is over a surplus of $S = [V^c(y, a, b, \delta_h) - V^d(y, a, b) + b - \kappa ph] > 0$.

Suppose the lender modifies the loan balance to \hat{b} such that the owner receives a fraction ϑ of the surplus S over his outside option, i.e.

$$V^c(y, a, \hat{b}, \delta_h) = V^d(y, a, b) + \vartheta[V^c(y, a, b, \delta_h) - V^d(y, a, b) + b - \kappa ph]$$

An analogous expression would hold for the lender.

Clearly, foreclosure would be avoided if both agents received a positive fraction of the surplus. If there were no gains from renegotiation, the mortgage would not be modified and foreclosure would obtain.

While modification is clearly beneficial from a pecuniary externality perspective (as it averts a foreclosure), it does not generally allow the lender to recoup the entire outstanding loan amount. Thus, the gains from renegotiation are that socially inefficient foreclosures are avoided, and lender loss is partially mitigated. One could therefore interpret the deadweight costs associated with default as arising in part due to *frictions* that do not permit renegotiation. This is why I alternatively refer to it in this article as a renegotiation inefficiency.

Frictions affecting the renegotiation process

There might be various factors that hinder efficient mortgage renegotiation in practice, such as asymmetric information⁶, debt securitization or capacity constraints (see e.g. Adelino et al. [2013], Agarwal et al. [2011, 2017]). In the latter two cases, inefficiency

⁶Adelino et al. [2013] argue that the possibility of self-cure of delinquent borrowers, risk of re-default following modification and moral hazard effects of modification on default behaviour account for low private mortgage renegotiations.

might arise due to a hold-up problem. In the case of securitization with multiple creditors, the value from holding out would increase if other creditors renegotiate their debt, hence this might delay or even prevent socially efficient renegotiation (see e.g. [Gertner and Scharfstein \[1991\]](#)). With capacity constraints interpreted as transaction costs, [Anderlini and Felli \[2001, 2006\]](#) show that *ex ante* transaction costs borne by agents can be interpreted as a form of hold-up, which might prevent socially efficient renegotiation. However, if transaction costs are borne *ex post*, then renegotiation is socially efficient. In section 1.7.1, I describe a simple extension of the model to allow for costly renegotiation, and consider whether renegotiation is inefficiently low.

1.3.5 Pecuniary externality associated with default choice

Pecuniary externalities operating through house prices contribute toward a wedge between an individual agent's and planner's choice of default. I note here that *any* individual choice that affects housing demand or supply would lead to pecuniary externalities in this environment.

Foreclosures, fire sales and the feedback loop

Consider first the pecuniary externality associated with default choice of owners. Relating the model above to traditional models of fire sales, a distressed owner has two options here: default or sale. Another difference with canonical fire sale models is that housing is indivisible, but the 'fire sale' channel is similar. A feedback loop is generated because defaults become more likely when prices decline as mortgage holders go underwater (which in turn drives prices down further upon foreclosure), and collateral constraints for homebuyers become tighter. Like canonical fire sale models, one requires an initial shock to income that affects homeowners and induces terminations.

Distributive and collateral externalities: intuition

Distributive externalities in this environment arise when changes in net worth due to asset price changes affect buyers and sellers (including lenders here) of the asset differently. With complete markets, changes in net worth would not matter as agents could fully insure themselves (cf. [Dávila and Korinek \[2018\]](#)), and the market clearing

condition would imply that asset price changes would have zero aggregate impact. However, whenever agents face incomplete financial markets (due to uninsurable idiosyncratic risk), they generally cannot fully insure against price movement-induced net worth changes. If they also face transaction costs, the market clearing condition does not net out. Therefore, distributive externalities do not wash out in the aggregate.

The sign of distributive externalities is generally ambiguous. The planner would intervene to change asset prices so as to benefit the agents in the economy who have higher marginal utility of consumption (the agents who are more constrained). For example, if sellers are more constrained than buyers, the distributive externality associated with default is negative. The planner would then choose to reduce default intensity in order to raise prices and therefore benefit sellers as opposed to buyers.

There are various articles that rely on distributive pecuniary externalities to motivate policy intervention (e.g. [Lorenzoni \[2008\]](#), [He and Kondor \[2016\]](#), [Itskhoki and Moll \[2019\]](#)). However, quantitative models studying normative policy generally assume representative agents and rely on collateral externalities to motivate intervention. As pecuniary externalities are small in the setting considered here, using distributive externalities to motivate policy intervention is not a practical drawback.

The *collateral* externality is common to that obtained in most of the literature (cf. [Dávila and Korinek \[2018\]](#)), and is due to the fact that individual decisions that impact asset prices also affect the borrowing constraints of other agents in the economy that are dependent on the value of the asset. Lower asset prices tighten the collateral constraint, which is not internalized by an agent when he chooses to default on his mortgage contract. Collateral externalities unambiguously favor fewer defaults and higher prices.

The price impact of a termination

The collateral and net worth terms for different types of agents, aggregated over the relevant state space, are all multiplied by the marginal house price impact of releasing an additional house on the market. This is represented by $\frac{\Delta p}{\Delta H}$, where :

$$H = \sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} \left(\sigma(y, a, b, \delta_h) + \delta(y, a, b, \delta_h) \right) * d\mu^O(y, a, b, \delta_h) \quad (1.20)$$

Externalities and house prices

As noted in section 1.3.4, foreclosure deadweight costs induce the planner to choose fewer defaults and foreclosures. However, this need not lead to higher prices as the planner is indifferent (from the perspective of limiting lender losses) between sale and repayment. From a pecuniary externality perspective, the planner would wish to choose repayment over sale if pecuniary externalities associated with sale⁷ and default are negative. This is because sales and defaults are equivalent in terms of their impact on housing supply, and promoting repayment has a positive impact on house prices. This implies that efficiency-based policy considerations about raising house prices in this environment depend solely on the magnitude of pecuniary externalities.

1.4 Numerical solution and calibration

The model has to be solved numerically, and the quantitative analysis studies the transition path following various shocks to steady state, as in Garriga and Hedlund [2020] and Chatterjee and Eyigungor [2015]. Details of the computational approach are described in appendix 1.9.2. First, the steady state of the model is calibrated in order to match certain moments of the U.S. mortgage and housing market prior to the Great Recession. Dynamics are introduced through the application of a combination of unanticipated and finite duration shocks to the steady state, and the price movements along the transition back to the steady state of the model are studied⁸.

A major objective of this paper is to evaluate mortgage default inefficiencies in the worst phase of a housing crisis that approximates the greatest observed price decline and foreclosure spike in the U.S. housing market during the Great Recession relative to the respective pre-crisis levels. Garriga and Hedlund [2020] argue that illiquid housing markets (modeled through search frictions in home sale and purchase that generate endogenous illiquidity), left tail income risk and credit tightening shocks can account for the downturn in the housing market.

⁷In this paper, I focus on the inefficiencies associated with default choices. The small magnitude of pecuniary externalities that I find makes the omission of sale choice externalities less relevant practically.

⁸An alternative approach could be to introduce aggregate risk, for e.g. allowing the Markov transition matrix to vary by the aggregate state of the economy. This approach is taken by Kaplan et al. [2020], Guren et al. [2018], but is not pursued in this paper.

The model in section 1.2 abstracts from modeling explicit housing search (although it features a sale transaction cost κ_h that is slightly higher than in most other housing models precisely to capture these frictions). Finally, the model is not equipped to induce a decline in the homeownership rate as observed during the Great Recession, which is constant due to the assumptions of non-convertibility between rental and owner-occupied housing and the absence of a construction sector.

1.4.1 Shocks applied to the steady state

All shocks are one-time and unanticipated. I follow the related literature (e.g. [Guren et al. \[2018\]](#), [Garriga and Hedlund \[2020\]](#) etc.) and consider a credit tightening shock modeled as a doubling of the downpayment fraction ι . I capture time-varying illiquidity in the housing market documented by [Garriga and Hedlund \[2020\]](#) through a one-period increase in the buyer transaction cost κ_b .

Countercyclical left skewness of the earnings distribution in the US was empirically established by [Guvenen et al. \[2014\]](#). One method to incorporate this skewness is to modify the Markov transition probabilities of the persistent income component (as in [Garriga and Hedlund \[2020\]](#)) by increasing the probability that an agent earning a middle income level transitions to a lower income level, so as to match the employment decline witnessed in the Great Recession. However, this latter approach does not (in concert with the credit tightening shock) generate the desired price decline and foreclosure spike in my model, suggesting that the one-time shocks here and the frictional search process in [Garriga and Hedlund \[2020\]](#) underpin this result.

[Guvenen et al. \[2014\]](#) also show that there is no countercyclical increase in income risk, i.e. an increase in the standard deviation (s.d.) of the earnings process which was found by [Storesletten et al. \[2004a\]](#). However, other articles like [Bayer et al. \[2019\]](#) find that income risk is countercyclical. In the model of section 1.2, the requirement that all non-owners pay rent constrains the magnitude of the increase in income risk that keeps non-owners' net worth positive. However, I find that increasing the s.d. of the persistent component of earnings to 0.19 (rather than 0.21 estimated by [Storesletten et al. \[2004a\]](#) for recessions) does not generate a significant price decline, even in concert with the credit tightening and buyer transaction shocks.

Given these findings, the exercise below models an income shock as a leftward shift of the income distribution, i.e. as a mean reduction. Whereas the mean of

the persistent component of the log income process is set to 0 in steady state, the one-period unanticipated income shock reduces the mean of the persistent component of the log income process to -0.01 . This is meant to capture an overall decline in earnings during a recession.

1.4.2 Calibration

The model is calibrated at an annual frequency. Most of the parameters are calibrated externally, and others are internally (jointly) calibrated in order to match certain moments in the data corresponding to 2007, which is the Survey of Consumer Finances (SCF) release date closest to the start of the crisis.

Externally calibrated parameters

The parameter values taken from the literature are displayed in Table 3.1. The utility function employed is:

$$u(c, \chi h) = \frac{\left(c^\tau (\chi h)^{1-\tau} \right)^{1-\gamma}}{1-\gamma}$$

The coefficient of relative risk aversion γ is set to 2, which is standard in the literature. τ is set at 0.8 to match the share of housing in consumption expenditure (see e.g. Davis and Ortalo-Magné [2011]). Agents are infinitely lived, and I choose the mortgage decay rate δ_m to yield a duration of mortgages of around 30 years, which is common in the U.S.

The (log) income process is calibrated following Chatterjee and Eyigungor [2015] and follows an AR(1) process with persistence $\rho_y = 0.97$ and standard deviation of residuals $\eta = 0.13$:

$$\log(y_t) = \rho_y \log(y_{t-1}) + \epsilon_t$$

There is no transitory income component in the baseline, for tractability⁹.

⁹As a robustness exercise, I added a transitory shock e which is log-normal with a standard deviation of 0.255 from Storesletten et al. [2004a]. Following Garriga and Hedlund [2020], I set the earnings process to be:

$$\ln(e \cdot y) = \ln y + \ln e$$

In a calibrated model that targets the same moments, I find that the magnitude of pecuniary externalities is similar to the case where earnings only have a persistent component.

Houses (owner occupied and rental) are assumed to be of fixed unit size. Transaction costs associated with buying (κ_b) and selling (κ_s) are set at 1% and 10% of the house value, following Chatterjee and Eyigungor [2015] and Garriga and Hedlund [2020]. Owners face a shock to house value (δ_h) that lowers their return. The shock takes two values, $\{0.01, 0.22\}$, based on Chatterjee and Eyigungor [2015], with probability $\{1 - \xi, \xi\}$ that shall be jointly calibrated. The value of the high depreciation shock is chosen to be consistent with the foreclosure loss from Pennington-Cross [2006], as in Chatterjee and Eyigungor [2015]. The probability of re-entry after default (θ) is chosen to be 0.25, implying an average exclusion period after default of 4 years.

The risk-free interest rate r is set to 2.5%. In the initial steady state pertaining to 2007 (prior to the recession), the maximum LTV (loan-to-value) ratio is set at 0.9, which is at the upper limit of the usual bounds of 0.8 and 0.9 that are commonly employed in the literature (see e.g. Corbae and Quintin [2015], Campbell and Cocco [2015]). Some other articles (e.g. Chatterjee and Eyigungor [2015]) do not include a LTV constraint and are better able to capture the lower end of the home equity distribution. The recovery fraction κ from foreclosure, i.e. the fraction of house value that a lender recovers from a foreclosure, is set at 78%, following the estimate in Pennington-Cross [2006], and is lower than the average foreclosure discount of 27% found by Campbell et al. [2011]. Rents are chosen so as to keep the rent-income ratio fixed at 0.2, following Guren et al. [2018].

The mortgage decay rate $\delta_m = 0.96$, in order to obtain a mortgage duration of 30 years, which is the average duration of a mortgage in the U.S.¹⁰.

Jointly calibrated parameters

The remaining parameters of the model are calibrated jointly in order to target the annual foreclosure rate, homeownership rate and the average LTV ratio from 2007. The annual foreclosure rate of 1.6% in 2007 is obtained from the National Delinquency Survey. The homeownership rate of 68% is obtained from the U.S. Census Bureau, while the average LTV ratio from the 2007 SCF provides a target of 0.62.

These moments are matched exactly using the following parameters respectively: the probability of a high depreciation shock (ξ), the housing preference parameter χ

¹⁰I solve for the value of δ_m that yields a half life of mortgage contracts of 15 years as follows. Based on the evolution of the loan balance, $b_n = \delta_m^n * b_0$, where b_0 is the loan balance at origination. Substituting for $b_n = \frac{b_0}{2}$ and $n = 15$ yields $\delta_m = 0.96$.

Table 1.1: Externally calibrated parameters

Parameter	Interpretation	Value	Source
γ	Coefficient of relative risk aversion	2	Standard
ρ_y	Persistence of earnings	0.97	Storesletten et al. [2004b]
η	Standard deviation of earnings	0.13	Storesletten et al. [2004b]
κ_b	Transaction cost coefficient of buying	0.01	Chatterjee and Eyigungor [2015]
κ_s	Transaction cost coefficient of selling	0.1	Chatterjee and Eyigungor [2015], Garriga and Hedlund [2020]
δ_h	Depreciation shock facing sellers	{0.01, 0.22}	Chatterjee and Eyigungor [2015]
r	Risk free interest rate	0.025	Arslan et al. [2015], Kaplan et al. [2020]
ι	Maximum LTV ratio	0.9	Campbell and Cocco [2015], Corbae and Quintin [2015]
κ	Foreclosure recovery fraction	0.78	Pennington-Cross [2006]
θ	Probability of re-entry after default	0.25	Chatterjee and Eyigungor [2015]
δ_m	Mortgage decay rate	0.96	Average mortgage duration = 30 years
τ	Consumption share in total expenditure	0.8	Average housing expenditure share

Table 1.2: Internally calibrated parameters

Parameter	Interpretation	Target moment	Parameter value	Moment value	Model value	Source
β	Discount factor	Average LTV	0.93	0.62	0.62	SCF 2007
χ	Housing preference parameter	Homeownership rate	8.62	68%	68%	U.S. Census Bureau
ξ	Probability of facing high depreciation shock	Foreclosure rate	0.103	1.6%	1.6%	National Delinquency Survey

and the discount factor β . The internally calibrated parameter values are displayed in Table 3.2.

The slightly large value of χ relative to the literature is because of the low mortgage decay rate, which implies a larger periodic mortgage payment, as well as the assumptions that all houses are of a single size, and therefore that owner-occupied and rental housing do not differ in terms of size. Chatterjee and Eyigungor [2015] use a mortgage decay rate of 0.988, which implies a mortgage duration of 80 years, and assume that owner-occupied housing is larger on average than rental housing, and therefore find a lower housing preference parameter. As utility from housing services depend multiplicatively on χ and h , one can interpret a value of 8.62 as a revised preference parameter value of $8.62/h$, which would, if the ratio of owner-occupied to rental housing size is assumed to be 1.5, be equal to 5.75. This revised value is similar to the calibrated value in Guren et al. [2018].

External validity

External validity is based on certain moments from the 2007 SCF, related to the distributions of owners and renters, financial wealth and income, and mortgage debt and income. The outcomes are shown in Table 1.3.

Broadly, the model fit is good, with the exception of the average financial wealth/average income ratio. This reflects partly the difficulty of generating enough precautionary

Table 1.3: Non-targeted moments (2007 SCF)

<i>Moment</i>	<i>Value</i>	<i>Model value</i>
Average income of owners/Avg. income of renters	2.15	2.08
Avg. financial wealth/Avg. income	1.83	1.17
Median mortgage debt/Median income	2.14	2.07
Fraction of agents with home equity ≤ 0.3	22.4%	20.1%

savings relative to the data due to a single source of (earnings) uncertainty, and partly due to the large mortgage decay parameter, which induces a larger mortgage payment and thereby lowers saving.

1.5 Results of the quantitative exercise

This section first discusses default and sale propensities of different types of owners, and then reports efficient default rates in the transitions exercise back to steady state following the combination of one-period shocks discussed above that are applied to the initial steady state.

I do not consider the welfare impact of an intervention here. Appendix 1.9.1 shows that there are small, positive welfare gains (measured in consumption equivalent variations) from the Planner’s intervention to alter default/foreclosure rates. These welfare gains are preserved even when the intervention is *anticipated*, as discussed in Appendix 1.9.1.2.

1.5.1 Default and sale patterns

One can express the default and sale choices of agents as functions of cash in hand ($= y + aR$) and loan balance b . The default results reported here are for agents facing a low depreciation shock. The *double trigger* is the result that defaulters tend to be owners with fairly large loan balances and adverse income shocks. On the other hand, when home equity is very negative (low), owners default because the cost of servicing the mortgage outweighs the benefits of being a homeowner. This is the *strategic default* hypothesis. Bhutta et al. [2017] find that strategic default is common among deeply underwater borrowers, while the double trigger is operative for less



Figure 1.2: Default and sale regions for low income owners.

Notes: *Sale (in red) is chosen by owners with moderate debt and low assets, while default (in blue) is chosen when owners are underwater. Figure drawn for owners facing low depreciation shock.*

underwater borrowers who have low cash-in-hand¹¹.

Default is generally chosen over sale when owners have negative home equity (net of transaction and depreciation costs), i.e. the mortgage is *underwater*. However, default must also dominate repayment, and default entails the penalty of exclusion from the owner occupied housing market for a random amount of time. Hence, underwater mortgage holders with low cash in hand would be more likely to default, as they have a high marginal utility of current consumption, and hence are more likely to discount future homeownership benefits. As home equity becomes more negative, some owners default despite having fairly large asset holdings. This reflects strategic default.

On the other hand, owners choose to sell when they have positive net home equity. Again, sale must also dominate the option to repay, and with the less stringent penalty for sale relative to default (sellers can access the homeownership market in the following

¹¹Recently, [Ganong and Noel \[2020b\]](#) find that 97% of defaults in their sample are liquidity-based, which they argue cannot be matched using existing quantitative models without default utility costs. Section 1.5.5 discusses why the motivation behind default choice does not affect the inefficiency results significantly.

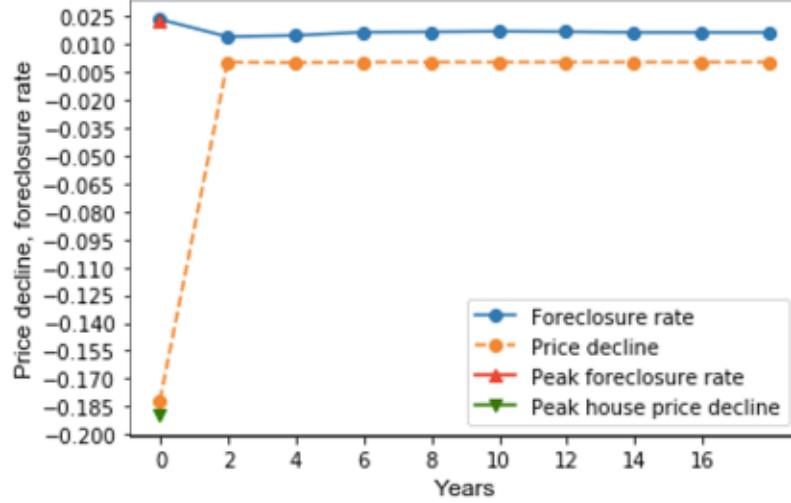


Figure 1.3: The decline in house prices and spike in the foreclosure rate along the transition back to steady state.

Notes: The economy moves back to steady state within 3 years, after having achieved a price decline of 18% and a peak foreclosure rate of 2.3%, close to the peak crisis levels of 20% and 2.23% respectively.

period), sale is chosen primarily when owners face a liquidity crunch. Both of these patterns are evident in Figure 1.2. Repayment occurs when agents have either fairly large asset holdings and low debt or when they have sufficiently large asset holdings relative to their debt.

1.5.2 Transition path and externalities

An objective behind the shocks chosen was to replicate the price decline and foreclosure spike observed in the worst phase of the Great Recession: the near 20% decline in prices and spike in foreclosure rate to a peak of $\approx 2.25\%$. As can be seen in Figure 1.3, the combination of shocks are successful in achieving both objectives. Given the one period shock duration, the economy converges back rapidly to steady state (in approximately 3 years). I do not capture the persistent decline in prices and increase in foreclosure rate during the downturn (seen in Figure 1.1), as my intention is to evaluate default inefficiencies in the worst phase of the crisis. In appendix 1.9.1.8, I show that applying a joint series of shocks for multiple periods can generate a slower transition to steady state (as in e.g. Garriga and Hedlund [2020]).

Table 1.4 shows the importance of the different shocks to capturing the peak

Table 1.4: Role of shocks to credit and income

	<i>Baseline</i>	<i>Exclude</i> ^a
Credit tightening shock		
Δ House prices	−19%	−8%
Δ Foreclosure rate	+7pp	+5pp
Income shock		
Δ House prices	−19%	−12%
Δ Foreclosure rate	+7pp	+2.5pp

^a The effect of a shock is the difference between the baseline and exclude columns

foreclosure-house price spiral. The *Exclude* column measures the marginal contribution of a shock by removing it and leaving the other shocks in place. The credit tightening shock contributes most significantly to the price decline while the income shock contributes most significantly to the foreclosure spike. The housing liquidity shock is the residual component and is less important quantitatively.

I solve for the constrained efficient default rate by incorporating the default wedge from equation (1.16). The results below for each component of the wedge correspond to the crisis period.

1.5.3 The nonpecuniary externality: incorporating lender loss from foreclosure

I now consider how default intensity changes upon incorporating lender losses from foreclosure. I do so by comparing the default rate in the decentralized equilibrium with the equilibrium default rate when a planner internalizes the lender loss from foreclosure while choosing whether to default¹².

Incorporating the foreclosure deadweight costs markedly affects the default intensity: the equilibrium default rate is 2.08% compared to the decentralized default¹³ rate of 2.3%. Foreclosure lender losses are significant due to the large price decline generated (18%), which lowers lender recovery from foreclosure sales. If foreclosure losses were actually larger in the downturn (perhaps due to countercyclical housing market illiquidity), then the constrained efficient foreclosure rate would be even lower.

The lower default rate upon internalizing lender losses is driven mainly by owners

¹²I briefly consider the case when the planner intervenes in successive periods along the transition path in appendix 1.9.1.

¹³The default rate is equivalent to the foreclosure rate in the model.

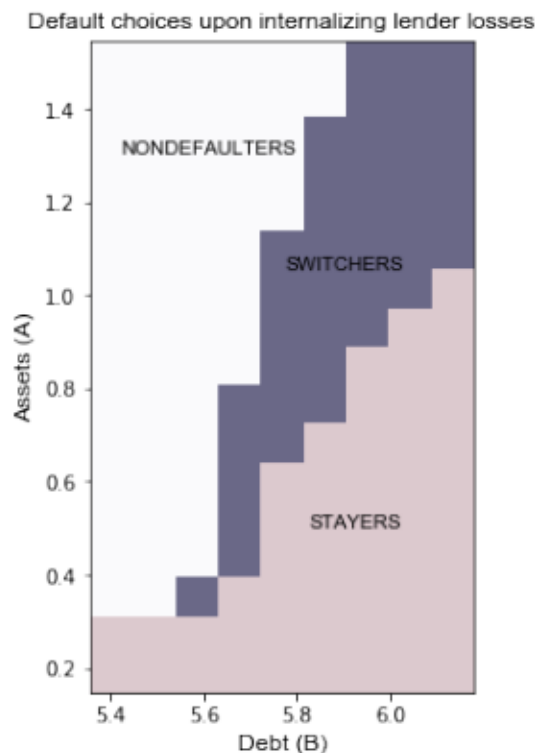


Figure 1.4: Default choices after internalizing foreclosure lender losses.

Notes: Some owners (in blue), called 'switchers', who default in the decentralized equilibrium do not upon internalizing lender losses. Owners with low assets (in pink), called 'stayers', default regardless of whether the externality is internalized or not.

with moderate to large asset holdings who are highly indebted, hence who contribute to larger lender losses. They switch from defaulting in the decentralized outcome to not doing so after internalizing the deadweight costs, as depicted in Figure 1.4.

1.5.4 Sign and magnitude of pecuniary externalities

Having solved for the equilibrium house prices and value and policy functions along the transition path in a manner described in appendix 1.9.2.1, I evaluate the PE expression in equations (1.18) – (1.19) using the approximation described in appendix 1.9.2.3.

The sign of the PE in the shock period may be positive or negative, and depends on the relative magnitudes of distributive and collateral externalities. If the sign of the PE associated with default is *negative*, it enters as an additional cost to be incorporated by the planner when choosing whether to default. Negative PE associated with defaults

are generally because the adverse impact of a marginal price decline (induced by a marginal default) on sellers (whose sale proceeds are lower), lenders (whose foreclosure recoveries are lower) and constrained buyers (whose collateral constraint becomes marginally tighter) outweigh the beneficial impact on unconstrained buyers (who pay less for a house). If the PE is positive, the explanation is reversed.

Using the approximation to evaluate the PE, I find that in the downturn, the PE associated with defaults is *negative*. This implies that the planner would choose to default less often relative to the decentralized equilibrium. Given the two expressions in equations (1.18) and (1.19), I can also decompose the PE into its distributive and collateral components.

I find that the demand side is quantitatively more significant (both for distributive and collateral externalities) than the supply (sellers and lenders) side. Owners who repay their instalment are favorably affected by lower prices as this reduces their maintenance payments, but they are not significant quantitatively.

1.5.5 Representing the pecuniary externality: the constrained efficient default rate

I now incorporate the pecuniary externality in the default choice of the Planner, and then compute the equilibrium default rate after internalizing the PE. Doing so, I find that the constrained efficient default rate is 0.15% lower in the shock period than the decentralized (competitive equilibrium) default rate. In other words, accounting for the PE would lead the planner to choose a default rate which is 2.296%. Hence, the PE is insignificant¹⁴.

Further, on the demand side, the distributive externality term for buyers opposes the collateral externality term. Indeed, if the constrained efficient default rate is based on the collateral externality alone, it is 0.6% lower than in the decentralized equilibrium (i.e. the constrained efficient default rate is now 2.29%).

¹⁴In appendix 1.9.1.8, I discuss how the result that foreclosure deadweight costs dominate (insignificant) pecuniary externalities holds even when the transition process is slower, i.e. the downturn is prolonged.

Why are pecuniary externalities small in the downturn?

One can understand the small magnitude of pecuniary externalities in the crisis by considering the expression for the pecuniary externality in equations (1.18) – (1.19).

Consider first the *distributive externality* term, fully specified in appendix 1.9.6.2. The distributive externality term for each agent type (buyers, sellers, lenders) is a weighted average of the marginal utilities across that type’s state space, where the weights are the densities of each agent-state. Many renters in the period of the shock choose to buy a house, because homeownership confers utility benefits and prices fall significantly, despite the tightening of credit. On the other hand, the price decline leads many owners to choose to default rather than sell¹⁵. The weighted average of marginal utilities for renters who buy a house is larger than the weighted average of marginal utilities for owners who sell their house, despite the fact that the mean consumption of buyers is higher than the mean consumption of sellers.

Furthermore, lenders (who possess the house upon default and foreclosure, and then sell it) are unconstrained and risk neutral (their marginal liquidity value is 1)¹⁶. Hence, the net impact on agents who supply housing is diminished, both because low prices lead to more foreclosures than sales, and the marginal liquidity value is lower for lenders than it is for sellers. This *insurance* property of default therefore dampens the redistributive motive underpinning efficient policy to correct distributive externalities. Finally, owners who make maintenance payments also benefit from price declines, thereby opposing sellers and lenders in the distributive externality term. Broadly then, distributive externalities in this environment are small and in favour of buyers¹⁷.

The other component of the PE is the *collateral externality*. Collateral externalities are important for the overall negative sign of the PE. However, endogenous loan

¹⁵The penalty for defaulting is temporary exclusion from the owner-occupied housing market, which could be considered weak, leading to excessive defaults over sales. I find that adding a default utility cost leads to fewer strategic defaults, but the magnitude of the distributive externality increases very slightly.

¹⁶In appendix 1.9.1, I consider how results change when the marginal liquidity value is parameterized to take values between 1 and 3.

¹⁷This conclusion should also hold if income shocks are left-skewed: even if skewness increases liquidity based defaults, the distributive externality expression does not distinguish between liquidity or strategic defaults, as the incidence is borne by unconstrained lenders in either case. Rather, it distinguishes between defaults and sales as the welfare impact from the latter is borne by constrained owners. Garriga and Hedlund [2020] use a frictional housing market model to argue that debt overhang led to fewer sales and more foreclosures in the housing bust.

pricing disincentivizes too many agents from leveraging up, which is not the case in the class of representative agent models referenced above. The presence of uninsurable idiosyncratic risk also introduces a precautionary saving motive that deters agents from borrowing large amounts. Hence, the overall magnitude of the pecuniary externality is small.

The price impact term measures the effect of an *additional* foreclosure on supply and house prices. Equations (1.18) – (1.19) suggest that the absolute magnitude of PE is increasing in $\frac{\Delta p}{\Delta H}$. Using a standard equilibrium housing model, I find that the price impact term is small. This is consistent with the empirical evidence in Campbell et al. [2011], Anenberg and Kung [2014]. However, Guren and McQuade [2020] show that introducing search frictions, credit rationing by lenders and differentiating between foreclosure/real estate owned (REO) and non-distress sales might amplify the effect of foreclosures on house prices beyond the standard supply channel¹⁸.

The extant literature on pecuniary externalities tend to find significant collateral externalities but do not typically have default risk or a precautionary motive. These models also consider short-term debt, hence (for a given loan balance) the amplification mechanism and fire-sale channel are stronger than in the case with long-term debt where payments are more spread out.

A large number of owners do not benefit or lose from price movements, aside from the maintenance payments. A natural extension studying the use of short-term collateralized credit should introduce an additional *collateral externality* and the overall magnitude of the PE associated with defaults would then be larger. This is explored in an extension in appendix 1.9.1.

1.6 Corrective policy

Corrective policy is meant to implement the constrained efficient default choice. This paper considers *ex post* policies that solely remedy default (and foreclosure) inefficiencies. Hence, the policy instruments are used only to yield the efficient default outcomes. The policymaker could induce agents to choose default efficiently through the use of various instruments. For example, default could be penalized through

¹⁸The foreclosure flag effect in their paper that dampens housing demand also features in my model.

the use of a tax on cash in hand, but such a policy is seldom observed in practice. Hence, I focus on incentivising repayment through the use of subsidies to net worth or reductions in the loan balance.

A recent article (Diamond et al. [2020]) has documented sizable non-pecuniary or social costs of foreclosure for borrowers. If accounted for by homeowners when choosing to default, these costs would tend to *lower* the private default surplus, defined as:

$$V^d(y, a, b) - \max\{V^s(y, a, b, \delta_h), V^c(y, a, b, \delta_h)\}$$

For a given loan balance, these non-pecuniary costs are more likely to induce either fewer defaults, or along with the inefficiencies derived earlier, lead to a social planner choosing to avoid default. Further, as the results below indicate, highly indebted distressed owners choose to default efficiently and hence do not receive debt reductions. The non-pecuniary, social costs described in Diamond et al. [2020] might also motivate debt reductions for these owners, beyond relief measures based on boosting their liquidity.

Another important point here is that the corrective *ex post* policy is individual-state specific, as in other heterogeneous agent environments (e.g. Davila et al. [2012]). This increases the informational burden of a policymaker. In appendix 1.9.1 I show (in the context of debt reductions) that a uniform policy that is not individual state contingent does not greatly affect outcomes.

The financing of the subsidy policies is not considered here. In particular, I am assuming that the various measures are non-distortionary. All results below pertain to the crisis period.

1.6.1 Debt reduction policies

These policies are aimed at modifying the loan balance so as to enable agents to make the efficient default choice. Hence, I report the debt reduction required to induce an owner who inefficiently chooses default to switch to repayment/continuation. As seen above, foreclosure deadweight costs generally dominate pecuniary externalities, both of which favour fewer defaults relative to the decentralized default rate. Hence, a salient policy is a loan balance *reduction*.

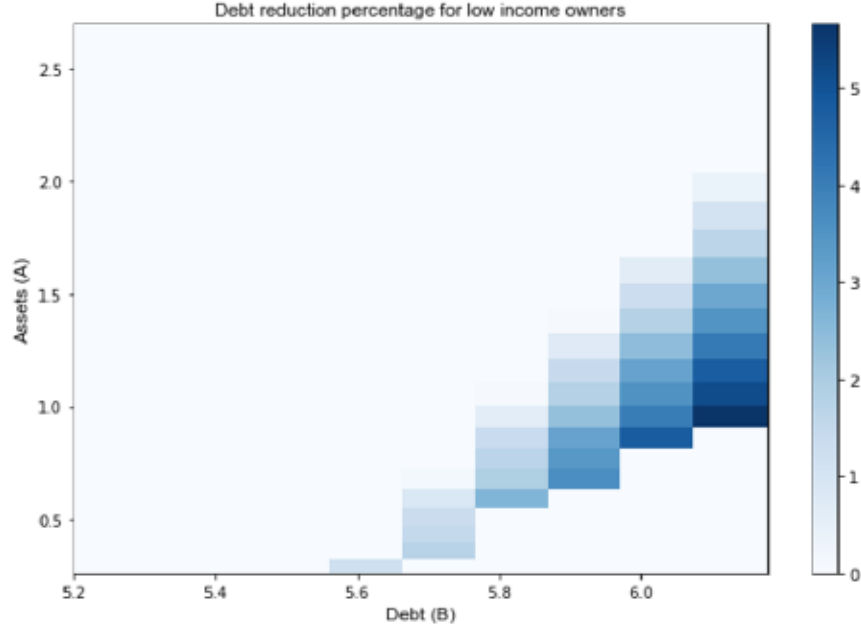


Figure 1.5: Percent debt reduction for low income owners.

Notes: This figure plots the variation of corrective debt reduction, expressed as a percentage of debt outstanding, across asset and debt levels. Lighter regions are areas with lower debt reductions..

The debt reduction policy then computes how much the loan balance (b) of an owner with state (y, a, b, δ_h) who inefficiently chooses default would have to be reduced in order to induce him to switch to repayment/continuation. Formally, I solve the following equation for \tilde{b} for an owner with state (y, a, b, δ_h) who inefficiently chooses default:

$$V^c(y, a, \tilde{b}, \delta_h) = V^d(y, a, b) \quad (1.21)$$

By the intermediate value theorem, one can find $\tilde{b}(y, a, b, \delta_h)$. The corrective debt reduction is then $1 - \frac{\tilde{b}(y, a, b, \delta_h)}{b}$.

The average debt reduction percentage for *inefficient* defaulters is approximately 3%, while the maximum debt reduction is around 6%. Figure 1.5 shows the variation of the debt reduction fraction for low-income owners with debt and asset holdings. For defaulters with low liquidity and highly negative home equity, the constrained efficient decision is to default: hence these agents do not receive a debt reduction. As is intuitive, the debt reduction is larger for inefficient defaulters with higher loan balance outstanding and lower asset holdings.

Comparison with HAMP policy

To relate the corrective policy above to actual policies implemented, consider the HAMP Principal Reduction Alternative (PRA) policy aimed at bringing the payment to income (PTI) ratio down to 31% or the LTV down to a target level. The average debt reduction in the HAMP Principal Reduction Alternative (PRA) program sample is 28% (Scharlemann and Shore [2016])¹⁹.

I find that the average payment to gross income ratio among defaulters in the crisis period is 58%, which is higher than the corresponding average PTI of 47% in the HAMP PRA sample of Scharlemann and Shore [2016]. On account of the downpayment requirement, the average LTV in the crisis period is 112%, considerably lower than the corresponding average in the HAMP PRA sample.

I compute the average debt reduction required to attain the HAMP PRA objective of reaching a 31% PTI ratio and find that it is approximately 28%, which is very similar to the average Principal reduction in Scharlemann and Shore [2016], but considerably greater than the 3% average corrective debt reduction.

Hence, the average corrective loan balance reduction required to implement the constrained efficient default choice is much smaller than the loan balance reduction required to implement the HAMP objective. Indeed, inefficiency correction accounts for about 10 percent of the average debt reduction under the HAMP PRA.

One can relate the state-contingency of debt reductions partially to HAMP modifications varying by pre-modification PTI and LTV ratios. Scharlemann and Shore [2016], Ganong and Noel [2020a] report that agents with high pre-modification PTI (> 0.31) and LTV ratios (reflecting higher pre-modification debt and/or lower income) received higher payment and principal reductions. Although the *efficient* debt reduction varied by owner assets as well, one can interpret this as partial evidence favouring the result that debt reductions are higher for defaulters with low income and high debt.

1.6.2 Subsidies to cash in hand

This subsidy is analogous to a debt reduction policy in terms of finding the increment to cash in hand that induces owners to make the efficient default decision. Thus,

¹⁹Ganong and Noel [2020a] document that the average *monthly* payment reduction under HAMP was 38%.

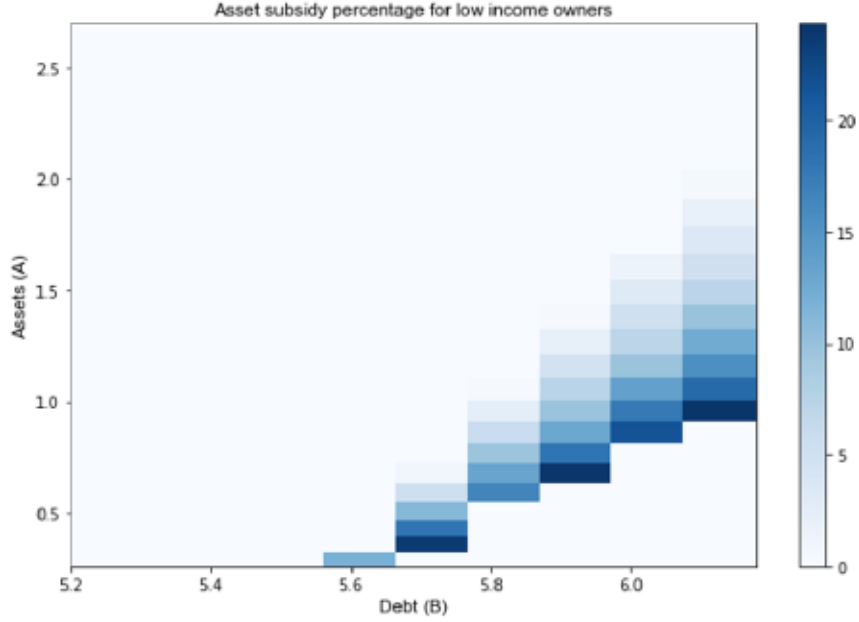


Figure 1.6: Percent asset subsidy for low income owners.

Notes: This figure plots the variation of corrective subsidy, expressed as a percentage of total assets, across asset and debt levels. Lighter regions have lower asset subsidies.

consider a policy that augments cash in hand for an owner with state (y, a, b, δ_h) who chooses default inefficiently through an asset subsidy (which can easily be interpreted as an income subsidy after dividing by R):

$$V^c(y, \tilde{a}, b, \delta_h) = V^d(y, a, b) \quad (1.22)$$

Again, the value function V^c is continuous in a , so a solution \tilde{a} to the above equation exists through the intermediate value theorem. The corrective asset subsidy is then $\frac{\tilde{a}(y, a, b, \delta_h)}{a} - 1$.

The subsidy varies considerably over the state space for agents who default inefficiently on their mortgages, from Figure 1.6. As with debt reductions, the efficient default choice for highly indebted owners with low liquidity is the same as private default choice. The subsidy is larger for highly indebted agents with low cash in hand (it is almost 20% for low asset values) and decreases as asset holdings rise, as is intuitive. The average asset subsidy (conditional on defaulting and receiving a subsidy) is approximately 13%.

1.7 Costly renegotiation and a discussion of *ex ante* policy

This section considers an extension to the baseline model of section 1.2 incorporating costly renegotiation. It also contains a brief discussion on *ex ante* policy in this environment. Appendix 1.9.1 considers two more extensions regarding the introduction of short-term collateralised borrowing by homeowners, and on the impact of changing the marginal lender liquidity value.

1.7.1 Renegotiation frictions: adding costly renegotiation to the model

In section 1.3.4, I showed that frictions to renegotiation would lead in part to inefficiencies (deadweight costs) associated with lender loss from foreclosure. In the model presented in section 1.2, this is because renegotiation is not permitted. However, it is easy to see that allowing for costly renegotiation would *partially* mitigate the losses to lenders from foreclosure. This is because renegotiation is voluntary, hence lenders should earn at least as much from renegotiating debt as they would from a foreclosure.

Here, I consider the extent of socially inefficient foreclosures in the presence of *ex ante* renegotiation costs in a simple environment where the division of the surplus from renegotiation is predetermined. Hence, I explore the impact of a specific friction (*ex ante* renegotiation costs) on efficient renegotiation intensity. I do not consider pecuniary externalities in this section.

Suppose the surplus from continuing with the mortgage plan is S and the lender receives a fraction α of the surplus but incurs a renegotiation cost \hat{c} *ex ante*, hence his payoff from renegotiation is $\alpha S - \hat{c}$. This can be interpreted as a screening cost to determine renegotiation eligibility²⁰.

For renegotiation to be efficient, $S - \hat{c} > 0$. Hence, inefficiency obtains if the lender's payoff from renegotiation is negative, which would arise if α is small enough. If

²⁰Agarwal et al. [2017] in their study of HAMP modifications mention how various eligibility criteria must be checked by servicers prior to a loan modification. Such screening costs are also incurred in private modifications.

the renegotiation cost were incurred *ex post* however, the lender's payoff would be $\alpha(S - \hat{c}) > 0$ and there would be no inefficiency.

I make the following simplifying assumptions for tractability. Renegotiation keeps the borrower indifferent between renegotiating debt and his outside option of default. The lender alone is also assumed to incur a transaction/renegotiation cost (ρ_{rc}) *ex ante* that is treated as a fixed cost. The outcome of renegotiation is a loan balance modification, and the defaulter who is renegotiated follows the mortgage plan defined in section 1.2 with the modified loan balance. If renegotiation is unsuccessful, the owner is foreclosed.

Let $\hat{b}(y, a, b, \delta_h)$ denote the modified loan balance that is the solution to the following equation:

$$V^d(y, a, b) = V^c(y, a, \hat{b}, \delta_h)$$

and $V^r(y, a, \hat{b}, \delta_h) = V^c(y, a, \hat{b}, \delta_h) = V^d(y, a, b)$. The loan pricing function becomes $Q(y, a', \hat{b})$, where one now accounts for the possibility of future loan modification in the expression for the current loan price. The lender chooses renegotiation if the net return from doing so exceeds the recovery amount from foreclosure: $Q(y, a', \hat{b})\hat{b} - \rho_{rc} \geq \kappa ph$.

I calibrate this extension in a manner described in appendix 1.9.1.5. I then compute the extent of social inefficiency when the steady state of this model extension is subjected to the combination of shocks described in section 1.4.

Illustrative results

I focus on socially inefficient foreclosures, which arise when renegotiation is not chosen despite there being gains from doing so. In other words, a foreclosure is socially inefficient if there is a surplus from renegotiation but foreclosure is chosen instead by agents.

I report the share of total defaults leading to foreclosure that would be renegotiated in the socially optimal benchmark. This captures the fraction of defaults that are inefficiently foreclosed in the decentralized equilibrium.

In the simulated downturn, the private renegotiation intensity is socially efficient, i.e. there are effectively no inefficient foreclosures. This is unsurprising for two reasons:

1. House prices decline considerably during the bust, which diminishes the recovery

amount for lenders from foreclosure. Hence, private renegotiation is chosen more frequently.

2. The baseline extension effectively gives all of the bargaining power to the lender. Let \tilde{b}_L be the loan modification that keeps a lender indifferent between renegotiation and foreclosure, i.e. solves $Q(y, a', \tilde{b}_L)\tilde{b}_L - \rho_{rc} = \kappa ph$. Similarly, \tilde{b}_B is the loan modification that solves $V^d(y, a, \tilde{b}_B) = V^c(y, a, \tilde{b}_B, \delta_h)$, which is the baseline mentioned above. Typically, $Q(., ., b')b'$ is increasing in b' , so for high values of debt, it exceeds the foreclosure recovery amount. Hence, the value of \tilde{b}_L that keeps lenders indifferent between renegotiation and foreclosure is generally lower than \tilde{b}_B . In the baseline where the lender holds the borrower to his outside option, the lender earns a larger share of the surplus and renegotiation is more likely to be chosen in the decentralized equilibrium. Hence the lender's decision to renegotiate would be close to the socially efficient choice.

As borrowers appropriate a greater share of the surplus, the participation constraint for the lender in the presence of *ex ante* renegotiation costs is more likely to be violated despite the presence of gains from renegotiation, and hence renegotiation is likely to be inefficiently low as borrowers have greater bargaining power.

I investigate this by computing the extent of inefficiently low renegotiation when the *borrower* has complete bargaining power, i.e. the outcome of the renegotiation process is \tilde{b}_L and therefore the borrower appropriates the entire renegotiation surplus. I find that around 9.3% of defaults that led to foreclosure would instead be renegotiated in the socially efficient benchmark.

These illustrative results suggest that introducing costly renegotiation might lead to inefficiently low renegotiation intensity. The (positive) gap between efficient and private renegotiation intensity rises as lenders receive a smaller share of the renegotiation surplus from renegotiation. By impeding efficient renegotiations, these frictions also exacerbate lender losses following default. Hence, *ex ante* renegotiation costs motivate a role for policy promoting loan modifications (renegotiation).

Table 1.5: Macroprudential debt taxes: outcomes relative to corresponding outcome without tax

Outcome	Δ in value: 1% debt tax	Δ in value: 5% debt tax
Mean steady state borrowing amount (%)	-3.2	-16.6
Foreclosure rate increase in transition (percentage pt.)	-1.8	-9
House price decline in transition (percentage pt.)	-0.25	-1
Welfare of owners in transition (%)	0.02	0.33
Welfare of renters in transition (%)	0.07	0.17

1.7.2 Constrained efficient *ex ante* policy, debt choice and externalities

In the model, a larger loan at origination makes agents more vulnerable to negative shocks. This makes forced sales or defaults more likely, and the resulting price decline in turn affects other homeowners and new buyers in the economy: this externality is not internalized by a buyer when he chooses his mortgage balance. Higher loan balances also negatively affect lender losses in the event of foreclosure. Hence, the inefficiency wedge features both deadweight costs and pecuniary components.

The major difficulty in evaluating efficient macroprudential policy is the multi-period nature of the mortgage loan. A higher loan balance at origination is consequential if the agent is forced to default at *any* point during the loan repayment program²¹. Hence, I focus on deriving *ex post* efficient policy.

Prudential debt taxes, foreclosure outcomes and welfare in the bust

I consider the impact that the imposition of a 1% and a 5% tax on debt *ex ante* has on foreclosure, house prices and welfare during a simulated crisis (bust)²². In each case, I calibrate the stationary equilibrium of the model with the given tax on debt to match the 2007 foreclosure and homeownership rates, as in section 1.4. I then simulate a downturn using the shocks as described in section 1.5.

The results for various outcome variables are in Table 1.5. Comparing outcomes across the two tax rates, note that higher macroprudential taxes improve outcomes during the crisis more. This is mainly due to fewer agents borrowing large amounts when *ex ante* debt taxes are higher, which makes them less likely to default during the simulated crisis.

²¹Appendix 1.9.1.2 discusses moral hazard induced overborrowing when policy interventions are anticipated *ex ante*.

²²One could alternatively consider higher down-payment requirements.

For a given tax rate, the mean borrowing amount in steady state with the debt tax is considerably lower than the corresponding level in a steady state without debt taxes, which is intuitive. Relative to steady state, both the *increase* in the foreclosure rate and the *decline* in house prices (expressed in percentage points (pp)) during the crisis are smaller with a macroprudential tax.

One can also compute the welfare gains during a crisis from the imposition of macroprudential policy, using the methodology described in appendix 1.9.1.1. Thus, I evaluate the percentage permanent increase in consumption that agents would require so that their welfare in the transition (the crisis period following shocks to steady state) *in the absence of macroprudential policy* is the same as their welfare in the transition with an *ex ante* tax on debt. Table 1.5 shows that aggregate welfare (in consumption equivalent variations) increases for owners and renters from the imposition of macroprudential policy. Hence, this exercise illustrates that the imposition of macroprudential policy improves the welfare of agents during a crisis.

1.8 Conclusion

This paper describes and evaluates the inefficiencies associated with mortgage default choices using a quantitative model of the housing bust in the United States during the Great Recession. It relates the normative literature motivating policy intervention to correct friction-based inefficiencies to the positive literature on housing market policies during the crisis. It extends the former by describing inefficiencies associated with a discrete choice in a model with an indivisible asset and long term debt when a policymaker intervenes *ex post*. It thereby provides a normative foundation for the design of crisis policies in the housing market.

A significant default inefficiency arises due to foreclosure deadweight costs for lenders. Renegotiation ameliorates deadweight costs, but might be inefficiently low. *Ex ante* renegotiation costs borne by lenders might lead to inefficiently low renegotiation when the division of the surplus from renegotiation favors borrowers. A fuller numerical analysis that determines the renegotiation outcome as a solution to a bargaining problem is the subject of future research. On the other hand, pecuniary externalities (distributive and collateral) are found to be small during the crisis.

From a policy perspective, the paper focuses on *ex post* policies such as debt

reductions that implement the constrained default choice. The average magnitude of corrective debt reductions is small compared to corresponding policies enacted during the Great Recession. Deriving inefficiencies to motivate macroprudential interventions such as debt limits or taxes is a challenging problem that requires further study.

Finally, the paper also provides a quantitative method to evaluate pecuniary externalities when equilibrium prices are determined through market clearing without a closed form expression.

1.9 Appendix to chapter 1

The appendices include a welfare analysis of the gains from policy intervention, other extensions to the model, a description of the computational approach, proofs and derivations of propositions from section 1.2, a description of the updating operator for the distributions, and the social welfare function from section 1.2.

1.9.1 Welfare gains from intervention and other extensions

This appendix considers further extensions to the model of section 1.2 and a discussion of corrective policy. I first discuss welfare gains and their distribution across agents.

The first extension considers whether there are welfare gains when the policy intervention is anticipated by agents *ex ante*. The second extension briefly reports how results are affected when the Planner intervenes in successive periods along the transition path. The third extension considers the impact of a uniform debt reduction on efficiency measured in terms of misallocation. The fourth extension looks at the consequences of changing the marginal liquidity value of lenders, while the final extension adds short-term collateral constraints for owners. This appendix also contains details about the calibration of the model with costly renegotiation discussed in section 1.7.

1.9.1.1 Welfare gains from Planner intervention

I evaluate welfare changes from a one-time unanticipated Planner intervention using the approach described for heterogeneous agents models by Krueger et al. [2017]. Specifically, I compute the consumption equivalent variation welfare measure, the percentage permanent increase in consumption that agents in the shock period of the transition process *without* policy intervention would require (as a subsidy) in order to be indifferent to their equilibrium outcome along the transition process *with* a policy intervention. This captures the welfare gains from the Planner's intervention²³.

I do this for each agent type j , for e.g. owners, renters. Hence, apart from studying consumption equivalent measures by income or asset levels as in Krueger et al. [2017],

²³This excludes welfare improvements for lenders, whose foreclosure losses are the quantitatively dominant component of the default inefficiency wedge. Hence, the welfare measure discussed here does not represent overall gains to all agents from the intervention.

I study welfare gains from the Planner's intervention by agent type (as in [Kiyotaki et al. \[2011\]](#)).

Denote the value functions in the shock period ($t = 0$) for an agent j with state vector s^j in the Planner and decentralized equilibrium respectively by $\tilde{V}^j(s^j; t = 0)$ and $V^j(s^j; t = 0)$. Entering in to period 0, agent types j are their types in steady state.

The consumption equivalent measure $(g(s^j))$ for an agent j with state s^j solves:

$$\tilde{V}^j(s^j; t = 0) = V^j(s^j; t = 0) \left(1 + g(s^j)\right)^{\tau(1-\gamma)}$$

Here, $g(s^j)$ is the required percentage consumption compensation for an agent of type j for moving from a transition with policy intervention to a transition without policy intervention.

Clearly, the consumption equivalent measure varies across agent-types and I wish to obtain type-specific aggregates. In order to do so, I consider the distribution of agent types in the pre-crisis steady state. I then evaluate the welfare of these agents along the transition path using the type specific value functions in the shock period, which summarize all of the relevant information of the transition process. Specifically, I compute consumption equivalent welfare measures for agent-types, based on their types in the pre-shock steady state, who undergo the transition process following the shocks, and I do this for both the decentralized economy and the economy with intervention by the Planner (see e.g. [Kiyotaki et al. \[2011\]](#)). Hence, I fix the types of agents as per the steady state, and evaluate their welfare along the transition path.

Denote the *steady state* value functions for agent type j with state s^j by $V_{ss}^j(s^j)$ and the *steady state* distribution of agents of type j by μ_{ss}^j .

Following the method described in [Krueger et al. \[2017\]](#), one can obtain type-specific aggregate consumption equivalent measures as follows:

$$g_{ss}^j = \left\{ \frac{\int_{s^j \in S^j} V_{ss}^j(s^j) d\mu_{ss}^j(s^j)}{\int_{s^j \in S^j} V^j(s^j; t = 0) d\mu_{ss}^j(s^j)} \right\}^{\frac{1}{\tau(1-\gamma)}} - 1$$

Table 1.6: Consumption equivalent variations associated with intervention (%)

<i>Case</i>	<i>Owners</i>	<i>Renters</i>
Unanticipated intervention	0.01	0.02
Anticipated intervention	2	0.5

$$\tilde{g}_{ss}^j = \left\{ \frac{\int_{s^j \in S^j} V_{ss}^j(s^j) d\mu_{ss}^j(s^j)}{\int_{s^j \in S^j} \tilde{V}^j(s^j; t=0) d\mu_{ss}^j(s^j)} \right\}^{\frac{1}{\tau(1-\gamma)}} - 1$$

Simple manipulations of the two equations above yields the relevant consumption equivalent measure for welfare gains from Planner intervention during the transition:

$$\frac{1 + g_{ss}^j}{1 + \tilde{g}_{ss}^j} - 1 = \left\{ \frac{\int_{s^j \in S^j} \tilde{V}^j(s^j; t=0) d\mu_{ss}^j(s^j)}{\int_{s^j \in S^j} V^j(s^j; t=0) d\mu_{ss}^j(s^j)} \right\}^{\frac{1}{\tau(1-\gamma)}} - 1$$

Intuitively, as the initial (shock period) distributions with or without Planner intervention are the steady state distributions $\{\mu_{ss}^j(s^j)\}_j$, this method captures the welfare *gains* associated with policy intervention in the transition process for agent types j based on their steady state distributions.

Row 1 of Table 1.6 reports the results of this comparison for the baseline case when the intervention is not anticipated. There are welfare gains for all agents from intervention, which are small.

1.9.1.2 Anticipated interventions and welfare

Throughout this paper, I have assumed that the policy intervention is not anticipated by agents *ex ante*. One would like to know whether anticipating the intervention would preserve welfare gains from the intervention. The effects of anticipating a policy intervention operate principally through the loan price and borrowing channel, and therefore the distribution of agents that the Planner takes as given when he intervenes to alter the default rate.

Approach

The transitions exercise in section 1.5 assumes that the shocks are completely unanticipated. As I focus on policy interventions during the crisis (i.e. in the period that the shock is applied), one would require an approach where agents anticipate an intervention but not the shock itself. In other words, agents anticipate the possibility of a crisis policy intervention but do not know when it shall occur.

In order to do so, I solve for stationary equilibrium assuming that lenders internalize the possibility of a policy intervention which does *not* actually occur in steady state. The intervention occurs in the period of application of shocks to steady state, hence the setup corresponds to agents in steady state (prior to the shock period) anticipating an intervention in the period when the shocks hit the economy.

Overborrowing when interventions are anticipated

I find that mean borrowing is higher in steady state when an intervention is anticipated: the mean loan amount at origination is around 4.8% higher than in a steady state without an anticipated intervention. This is intuitive, and operates through the loan price channel: loan prices are higher as default risk is lower when the Planner is expected to intervene to reduce foreclosures. This corresponds to moral hazard and excessive risk-taking associated with *ex post* targeted²⁴ interventions, discussed in e.g. [Bianchi \[2016\]](#) and [Jeanne and Korinek \[2020\]](#). As those papers discuss, targeted *ex post* or crisis policy interventions typically need to be accompanied by macroprudential policy measures like debt limits or macroprudential taxes on debt to correct for such moral hazard-induced overborrowing.

Welfare gains from anticipated intervention

Considering welfare gains from an anticipated policy intervention, I compare the aggregate consumption equivalent variations along the transition process with and without the policy intervention when the intervention is anticipated. Hence, I measure the percentage permanent increase in consumption such that agents in the transition process who do *not* anticipate and face an intervention would have the same welfare

²⁴As section 1.3 discusses, the Planner chooses default for each individual owner-state.

as in the transition process *with* an anticipated intervention. The results are reported in the second row of Table 1.6. Welfare increases for owners and renters, and the magnitudes of welfare gains are higher than the case when the intervention is completely unanticipated (row 1).

1.9.1.3 Planner intervention occurs in the first two periods

Section 1.5.3 describes how foreclosure deadweight costs affects Planner default choice and the aggregate equilibrium default rate when the Planner intervenes only during the period of the shock.

One could also consider how the efficient equilibrium default rate in the shock period is affected when the Planner intervenes for multiple periods along the transition path. For concreteness, I consider the case when the Planner intervenes only in the first two periods along the transition path, i.e. the shock period and the period after. Allowing the Planner to intervene repeatedly in an *unanticipated* manner requires one to assume that the interventions of the Planner in the first two periods along the transition path are never anticipated by agents in the economy.

I focus solely on the deadweight cost inefficiency as the pecuniary externality is tiny. I find that the Planner's equilibrium default rate when the intervention occurs in the first two periods along the transition path is 2.075%, compared to the decentralized equilibrium default rate of 2.3% in the crisis period. Hence, the efficient equilibrium default rate with a one-time intervention is slightly *higher* than the corresponding default rate when the intervention occurs twice.

This occurs as a consequence of equilibrium house prices being slightly higher along the transition path following the shocks under the two-time intervention rather than the one-time intervention. Indeed, the economy converges back to steady state after 2 periods when the intervention is repeated. Higher expected future prices affect purchase (favorably) and sale (adversely) choices in the shock period, and this results in higher prices in the shock period. Higher prices also reduce foreclosure lender losses, but as private default choices are less frequent with higher prices, the overall default rate is lower when the Planner intervenes in successive periods.

Table 1.7: Misallocation under a uniform debt reduction policy

<i>Debt reduction fraction</i>	<i>Type 1 mean</i>	<i>Type 2 mean</i>
0.9	0.35%	0.43%
0.97 (mean optimal debt reduction for defaulters)	0.003%	1.3%
0.99	0.0026%	1.664%

1.9.1.4 A uniform debt reduction policy

The corrective debt reduction policy in the model environment with heterogeneous agents is individual state-specific, i.e. it depends on an owner's state vector (y, a, b, δ_h) . This clearly requires the policymaker to possess a lot of information about owners' states, which is often not the case in practice. A natural question is how our results would be affected if the policymaker could only impose a uniform policy for all agents. Such a policy would either be too generous for some owners, who would be subsidised more than the efficient individual specific debt reduction from equation (1.21) would require; or it would be insufficient for others, who would not receive a suitable debt reduction.

I evaluate the inefficiency arising due to the uniform policy as follows. I first solve for the efficient debt reduction policy using equation (1.21). Next, I choose the uniform fraction of debt that is reduced for all agents, and determine the default choices of owners who obtain this uniform debt reduction. I then compare the default choices of individual owners under the uniform debt reduction policy to their choices under the efficient debt reduction policy. Some agents who would have defaulted under the efficient policy now would not under the uniform policy (call them 'type 1' agents), and vice versa (call them 'type 2 agents'). I report the means of both types of agents, as a measure of the misallocation associated with a uniform policy. For e.g. a type 1 mean of 1% indicates that a fraction 0.01 of all owners would have defaulted under the efficient policy but do not under the uniform policy.

I consider three debt reduction fractions: the mean of the efficient debt reduction policy (for defaulters) and uniform debt reductions of 1% and 10%.

Table 1.7 shows that the extent of misallocation is quite small. Type 2 agents are more important quantitatively: they represent agents who did not default under the efficient policy but do so under the uniform debt reduction policy. As the uniform debt reduction becomes less generous, they become more significant quantitatively. This occurs because they do not receive a sufficient debt reduction to induce a switch to continuation/repayment. Type 1 agents generally become more significant as the debt

Table 1.8: Internally calibrated parameters in baseline extension with costly renegotiation

Parameter	Interpretation	Target moment	Parameter value	Moment value	Model value	Source
β	Discount factor	Average LTV	0.921	0.62	0.62	SCF 2007
χ	Housing preference parameter	Homeownership rate	10	68%	68%	U.S. Census Bureau
ξ	Probability of facing high depreciation shock	Foreclosure rate	0.13	1.6%	1.6%	National Delinquency Survey
ρ_{rc}	Renegotiation cost	Ratio of foreclosure to modification rates	0.48	0.67	0.7	Agarwal et al. [2017]

reduction becomes more generous, but not to the extent that type 2 agents become significant when the debt reduction fraction is smaller.

A uniform debt reduction policy could trade off the cost of a more generous policy against reduced type 2 misallocation: as the results indicate, more generous debt reductions lead to an overall decline in type 2 misallocation.

1.9.1.5 Calibrating the extension with costly renegotiation

Section 1.7.1 above considered a simple extension allowing for costly renegotiation. In this appendix, I discuss the calibration of this extension. Broadly, the baseline model targets the same moments and uses the same externally calibrated parameters as in section 1.4.

In order to calibrate the renegotiation cost, I target private modification activity based on the discussion in Agarwal et al. [2017], section IV.B. There, they show that in the pre-HAMP period (July 2008-March 2009), the ratio of private modifications to foreclosure activity is approximately 40%. As this period is after the year 2007 chosen to calibrate the internally chosen parameters of the model, I assume that the ratio of foreclosure rate to modification rate is 2/3. As the foreclosure rate is known to be 1.6% in 2007, and I assume that defaults are either foreclosed or renegotiated, this provides a target with which to calibrate the renegotiation cost parameter ρ_{rc} .

1.9.1.6 Parameterizing the marginal liquidity value of lenders

Lenders in the model above are unconstrained and risk-neutral, hence one cannot readily analyse the amplification of shocks through the financial sector in our environment. In models where lenders face the possibility of being financially constrained, foreclosure losses affect their balance sheets and would lead to reduced lending activity (see e.g. Guren and McQuade [2020] for a consideration of this channel).

Table 1.9: Variation of the wedge in default rates with marginal lender liquidity value

α	Wedge with PE	Wedge with deadweight cost inefficiency
1	0.15%	9.4%
1.25	0.151%	10.8%
1.5	0.153%	11.25%
2	0.4984%	12.5%
3	0.4984%	14%

However, one may still consider how higher marginal lender liquidity value would affect the results described above. A simple method is to parameterize lender liquidity value (α) to take values greater than unity. This could capture the possibility of binding capital constraints faced by lenders during a downturn, or could also reflect a higher weight by the Planner on lender payoffs (interpretable as favouring financial sector stability) in the social welfare function. Another possible way is to consider shocks to the lenders' stochastic discount factor, representing higher aggregate risk in the downturn.

Parameterizing lender liquidity value by α , both lender loss and the pecuniary externality term concerning lenders in appendix 1.9.6.1 would be scaled up by α . Hence, increasing α raises both the lender loss and lender pecuniary externality terms commensurately. Marginal liquidity value can be parameterized based on models where intermediaries face constraints, for e.g. [Gertler and Kiyotaki \[2015\]](#). That paper parameterizes marginal value of net worth to a banker in the steady state of their model to be equal to 1.9, which rises to almost 3 during a bank run. Hence, the range of α I consider is between 1 and 3.

Table 1.9 shows the difference between constrained efficient and decentralized default rates under different values of α . The first row is the baseline: when $\alpha = 1$, the overall difference between the constrained efficient default rate and the decentralized default rate is 9.55%. Hence, if the decentralized default rate is 2.3%, then the constrained efficient default rate is $0.9045 * 0.023 = 2.08\%$. Accounting for lender loss alone leads to a constrained efficient default rate that is 9.4% lower than the decentralized default rate. The remaining rows are interpreted similarly.

Notice that as α moves closer to 2, the Planner chooses a constrained efficient default rate that is almost 13% lower than the decentralized default rate. This is in keeping with the intuition that if lenders value liquidity more, then the social value of changes to their net worth would be magnified.

Moving now to the pecuniary externality, note that while the pecuniary externality is increasing in α , the impact on the efficient default rate is not very significant. Increasing α increases the magnitude of distributive externalities, but does not affect the collateral externality term. The discussion in section 1.5.5 indicates that collateral externalities ultimately are more important for the significance of the overall pecuniary externality, and this is corroborated by the results of the parameterization exercise.

1.9.1.7 Home equity collateral constraints

This section develops a point mentioned in section 1.5.5 by introducing an additional collateral constraint, a short-term borrowing constraint for owners collateralised by their house, and considers how the resulting additional collateral externality affects the overall significance of the pecuniary externality term.

Another reason for adding home equity constraints is to relate the paper to the literature on housing wealth effects, and the marginal propensity to consume out of housing wealth (see e.g. Berger et al. [2017]). If consumption out of housing wealth is a significant factor in the decision to own a house, then it is reasonable to consider how our results would be altered in a model that includes a simple short term collateral constraint.

I extend the model in section 1.2 to allow for a non-defaultable one period home equity constraint that can be issued by homeowners. Formally, the home equity constraint is:

$$a' \geq -\lambda_a p h \tag{1.23}$$

Thus, homeowners can now borrow up to a fraction λ_a of their home value, and this is assumed to be non-defaultable. This implies essentially that the home equity constraint is senior to mortgage debt, which is counterfactual but is chosen for reasons of tractability²⁵.

Adding the home equity constraint implies, as mentioned above, that there will be an additional collateral externality term in addition to equation (1.19). Following the

²⁵As discussed by Kaplan et al. [2020], allowing home equity constraints to be defaultable adds another endogenous loan pricing function, which adds to the computational complexity.

Table 1.10: Internally calibrated parameters in model with home equity collateral constraints

Parameter	Interpretation	Target moment	Parameter value	Moment value	Model value	Source
β	Discount factor	Average LTV	0.922	0.62	0.62	SCF 2007
χ	Housing preference parameter	Homeownership rate	9	68%	68%	U.S. Census Bureau
ξ	Probability of facing high depreciation shock	Foreclosure rate	0.113	1.6%	1.6%	National Delinquency Survey

same derivation as in appendix 1.9.6, this term is:

$$\sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} \eta_a^o(y, a, b, \delta_h) * \lambda_a * d\mu^O(y, a, b, \delta_h) * \frac{\Delta p}{\Delta H} \quad (1.24)$$

If the home equity constraint does not bind for the relevant agent, the Lagrange multiplier on the home equity constraint for an owner (η_a^o) is equal to zero.

The model with home equity constraints is calibrated to match the same set of moments as in section 1.4. The externally calibrated parameters are as in Table 3.1, while the internally calibrated parameters are now displayed in Table 1.10.

As hypothesized, the addition of home equity constraints makes a significant difference to the magnitude of the PE term. In particular, the Planner now chooses a default rate that is 2 basis points lower than the decentralized equilibrium default rate: the constrained efficient default rate due to the PE alone is 2.28%

1.9.1.8 Generating a prolonged downturn: applying shocks for multiple periods

The results in this paper are for a *one-time* unanticipated joint series of shocks. As Figure 1.3 shows, the economy recovers from the crisis and goes back to steady state within 3 years. However, Figure 1.1 showed that the foreclosure-house price spiral was persistent and it took much longer for these measures to revert to pre-crisis levels.

I now follow Garriga and Hedlund [2020] in applying a joint series of shocks for a longer span of time in an attempt to obtain a slow recovery. Here, I show results when the income shock and housing liquidity shock are each applied for two consecutive periods. Figure 1.7 shows that these two shocks are sufficient to generate a nearly 20% decline in house prices and a foreclosure rate of 2.45%. Hence, the downturn is more severe when this combination of shocks is applied for two consecutive periods.

From an inefficiency perspective, I find that pecuniary externalities (PE) remain

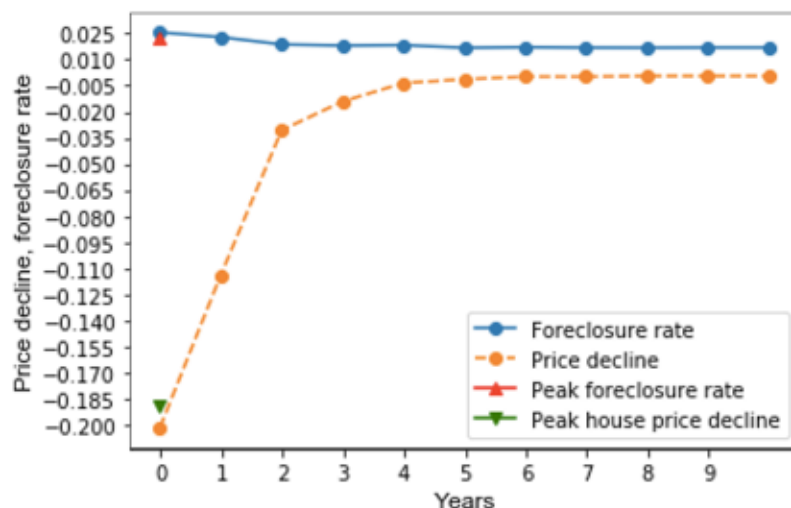


Figure 1.7: The decline in house prices and spike in the foreclosure rate along the transition path following a combination of shocks applied for two periods.

Notes: The economy moves back to steady state within 6 years, after having achieved a price decline of 20% and a peak foreclosure rate of 2.45%.

insignificant when the transition process is prolonged: accounting for PE alone does not change the foreclosure rate from the competitive decentralized equilibrium.

Further, the foreclosure deadweight cost is the dominant inefficiency: the foreclosure rate that accounts for this inefficiency alone is 2.21%, compared to a decentralized foreclosure rate of 2.45%.

Thus, the focus in the paper on evaluating inefficiencies at the *peak* of the foreclosure-house price spiral does not affect results as opposed to considering a more persistent housing downturn (a slower transition back to steady state). The assumption that the policy intervention is unanticipated by agents is more reasonable when the shocks are of single-period duration: if agents know that the crisis shall last for two periods and in the initial crisis period anticipate a policy intervention in the subsequent period as well, then it warrants an analysis similar to appendix 1.9.1.2.

1.9.2 Computing the constrained efficient solution

1.9.2.1 Solving the quantitative model

I solve the model using value function iteration, owing to the discrete choices of ownership, default and sale. The income process is discretized using the Rouwenhorst method with a five state Markov chain. The asset and loan balance grids are discretized using grids with 61 points each²⁶. The approach is to first solve for the decentralized stationary equilibrium of the economy, and I jointly solve for the stationary value and policy functions and the stationary distributions using standard algorithms (see e.g. Heer and Maussner [2009]). The Planner's stationary equilibrium is determined by jointly solving for the policy functions, stationary distributions, price responsiveness using the slope of the demand curve and the pecuniary externality term from equations (1.18) – (1.19) in a manner described below. In particular, the policy functions and distributions (which are defined on a discrete array grid) during the iterative process are used to compute the pecuniary externalities and the slope of the demand curve in each iteration until convergence. In order to account solely for the nonpecuniary externality, I follow the same procedure but allow for the Planner to internalize the lender loss from foreclosure, as in equation (1.16).

Transition dynamics are solved by adapting the algorithm in Rios-Rull [1997] and Garriga and Hedlund [2020] suitably to the model presented above. Specifically, the initial distributions of owners, renters and flagged agents are obtained from the steady state of the model, and the economy is assumed to converge back to steady state following the unanticipated one-time shocks in $T = 31$ periods. The period T value and policy functions are obtained from the steady state solution, and I solve for the period value functions in $t = 0, \dots, T - 1$ using backward induction. In order to obtain the period $t > 0$ distributions, I first obtain the equilibrium price p_t^* and period t policy functions by evaluating policy functions and hence excess demand in period t on a fine grid of prices, using the value and policy functions in $t + 1$ obtained from backward induction. The equilibrium price p_t^* clears the market by setting excess demand equal to zero. At this equilibrium price, I then update the distributions as described in appendix 1.9.3. Convergence is checked using two criteria: the price vector $\{p_t\}_{t=0}^{T-1}$ does not vary sizably between iterations (i), i.e. $\sup_t |p_t^i - p_t^{i-1}| < \epsilon^p$;

²⁶I have considered finer grids for robustness and found that they do not affect the results significantly.

and the final period T distributions do not differ significantly from the steady state distributions, i.e. $\max_{j \in \{O, R, aut\}} (\sup |\mu_T^j - \mu_{ss}^j|) < \epsilon^\mu$.

In order to compute the wedge in the period of the shock to the stochastic steady state, given the stationary distributions computed as mentioned above, I use the formula for the wedge given in equation (1.16), and approximate the pecuniary externality components in a manner described below. Having obtained the wedge, I can compare the default choice of an individual owner with state s^o to that of the Planner for the same state, accounting for the wedge. The difference in default rates under the Planner and the decentralized solution is then obtained by aggregating the individual and Planner default choices across the distribution of owners.

1.9.2.2 Approximating price responsiveness

A key part of the PE term is $\frac{\Delta p}{\Delta H}$, the price impact of an additional unit supplied on the market. There is no closed form expression for house price determination, a feature shared with other models where asset prices are determined through a market clearing condition. Hence, approximation is required.

This is done by calculating the slope of the housing demand curve at the competitive equilibrium price. In the transitional dynamics algorithm, the equilibrium price in each period is computed by solving for the policy functions at each guessed price in a price grid, and using them to obtain excess demand corresponding to each price. The equilibrium price is then the grid price that sets excess demand to zero in that period. With a fine enough price grid, there is considerable variation in the policy functions and particularly housing demand with prices. I interpolate a 'smooth' housing demand function (a curve) along the price grid using a cubic spline, and then choose a small enough interval around the equilibrium price with which to calculate how the equilibrium price would change along the demand 'curve'. The price impact is then the inverse of the slope of the housing demand function at the equilibrium price.

1.9.2.3 Approximating the pecuniary externality

Transitional dynamics determine the *competitive equilibrium solution* to the model of section 1.2 following a shock to the stochastic steady state. The ultimate objective is to compute the constrained efficient solution along the transition path.

One can evaluate pecuniary externalities in steady state, interpreted as a perturbation induced by marginally changing some parameter that induces more foreclosures (e.g. the transaction cost for sellers). A feature of the PE expression in equations (1.18–1.19) that affects the Planner’s choice of default is that the policy functions, price impact *and* the externality term should be determined jointly. This is computationally difficult, but can be done in steady state. Doing so, I find that steady state pecuniary externalities associated with default choice are *positive* but insignificant and do not lead to Planner default choices that vary significantly from the decentralized outcome.

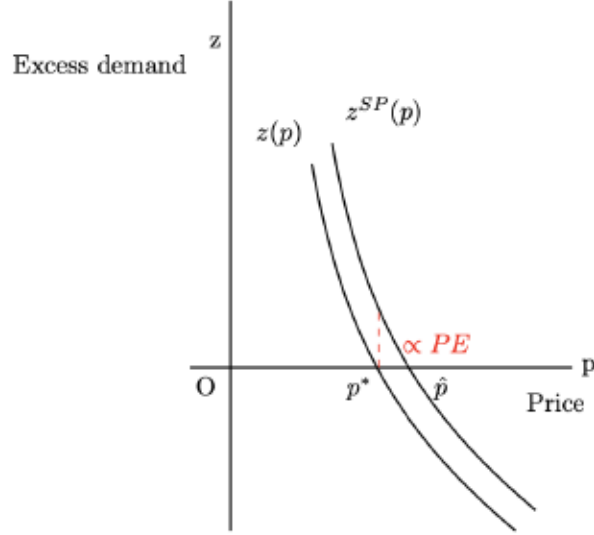
Ideally, one would carry out a similar exercise to evaluate pecuniary externalities along the transition path. However, this approach is difficult to carry out when considering transitions following a shock, so one is interested in finding an approximate representation of how the pecuniary externality would affect agent’s default choices if they internalized them. It is important to use the appropriately defined value functions in order to compute the approximate pecuniary externality. For example, if one is interested in just the PE component of the wedge associated with default choice and the constrained inefficiency that arises therefrom, one would use the value functions defined as in section 1.2.

Approximation: *If the pecuniary externality (PE) component of the wedge evaluated at the competitive equilibrium is not too large, then it is approximately equal to the PE based wedge evaluated at the Planner’s solution.*

This approximation is not likely to hold if the Planner’s solution differs considerably from the competitive equilibrium allocation, which would tend to occur if the PE were large in absolute value. Since the policy functions and distributions are functions of the asset price, and the asset price in equilibrium clears the housing market, the excess demand function for the Planner would tend to vary with the price and hence the PE term which depends on the equilibrium price and the policy functions and distributions would also tend to vary with the price. If the competitive equilibrium and Planner equilibrium prices are not too far apart, one can use continuity based arguments (following Stokey et al. [1989], Chatterjee et al. [2007]) using discrete choice *probabilities* instead of indicator variables to argue that the above approximation is reasonable.

What this approximation implies is that one could use the competitive equilibrium/decentralized solution along the transition path and the PE terms calculated

Figure 1.8: The pecuniary externality evaluated at the competitive equilibrium and the computational approximation



at the competitive equilibrium solution for each period in order to approximate the corrective policy and the Planner's choice. This is represented in Figure 1.8 and the following discussion provides a brief justification.

Going from the PE term to discrete choice probabilities

The first step of the approach laid out above is to use the policy and density functions evaluated at the competitive equilibrium, along with the PE terms evaluated using the same functions and for a given price impact ($\frac{\partial p}{\partial H}$), in order to represent the choice probabilities of the Planner.

Choice probabilities are derived from the discrete choices of agents by assuming that agents choose an option with a certain trembling hand possibility of error:

$$V_i + \epsilon_i \gtrless V_j + \epsilon_j$$

The *difference* in error terms ($\epsilon_i - \epsilon_j$) is then assumed to be drawn from the Type-1 extreme value (Gumbel) distribution. As is commonly known (see e.g. Train [2009]), this implies that the probability of choosing i is then given by $\frac{\exp(V_i)}{\exp(V_i) + \exp(V_j)}$.

In the context of the housing model of section 1.2, the private choice of repayment

over sale or default occurs if:

$$V^c(y_t, a_t, B_t, \delta_{ht}) \geq \max \left\{ V^s(y_t, a_t, B_t, \delta_{ht}), V^d(y_t, a_t, B_t, \delta_{ht}) \right\}$$

Now, the Social Planner's corresponding repayment choice differs from the private choice by accounting for the externality term, $PE(t)$:

$$\tilde{V}^c(y_t, a_t, B_t, \delta_{ht}) \equiv V^c(y_t, a_t, B_t, \delta_{ht}) + PE(t) \geq \max \left\{ V^s(y_t, a_t, B_t, \delta_{ht}), V^d(y_t, a_t, B_t, \delta_{ht}) \right\}$$

Let $P^s(y_t, a_t, B_t, \delta_{ht})$ denote the private sale probability, and $P^d(y_t, a_t, B_t, \delta_{ht})$ denote the private default probability for an agent with state vector $(y_t, a_t, B_t, \delta_{ht})$. Exploiting the distributional assumption, the probability of default (for example) can be written as:

$$P^d(y_t, a_t, B_t, \delta_{ht}) = \frac{\exp \left(V^d(y_t, a_t, B_t, \delta_{ht}) \right)}{\exp \left(V^c(y_t, a_t, B_t, \delta_{ht}) \right) + \exp \left(V^s(y_t, a_t, B_t, \delta_{ht}) \right) + \exp \left(V^d(y_t, a_t, B_t, \delta_{ht}) \right)}$$

Henceforth, tildes are used to denote the Planner's choice of variables. Hence,

$$\tilde{P}^d(y_t, a_t, B_t, \delta_{ht}) = \frac{\exp \left(V^d(y_t, a_t, B_t, \delta_{ht}) \right)}{\exp \left(\tilde{V}^c(y_t, a_t, B_t, \delta_{ht}) \right) + \exp \left(V^s(y_t, a_t, B_t, \delta_{ht}) \right) + \exp \left(V^d(y_t, a_t, B_t, \delta_{ht}) \right)}$$

Since $\tilde{V}^c(y_t, a_t, B_t, \delta_{ht}) \equiv V^c(y_t, a_t, B_t, \delta_{ht}) + PE(t)$, this can be written as:

$$\tilde{P}^d(y_t, a_t, B_t, \delta_{ht}) = \frac{\exp \left(V^d(y_t, a_t, B_t, \delta_{ht}) \right)}{\exp \left(V^c(y_t, a_t, B_t, \delta_{ht}) \right) \exp \left(PE(t) \right) + \exp \left(V^s(y_t, a_t, B_t, \delta_{ht}) \right) + \exp \left(V^d(y_t, a_t, B_t, \delta_{ht}) \right)}$$

Similarly,

$$\tilde{P}^s(y_t, a_t, B_t, \delta_{ht}) = \frac{\exp \left(V^s(y_t, a_t, B_t, \delta_{ht}) \right)}{\exp \left(V^c(y_t, a_t, B_t, \delta_{ht}) \right) \exp \left(PE(t) \right) + \exp \left(V^s(y_t, a_t, B_t, \delta_{ht}) \right) + \exp \left(V^d(y_t, a_t, B_t, \delta_{ht}) \right)}$$

One can derive the Planner's choice probabilities expressed in terms of the private choice probabilities as follows:

$$\frac{1}{P^d(y_t, a_t, B_t, \delta_{ht})} - 1 = \frac{\exp(V^c(y_t, a_t, B_t, \delta_{ht})) + \exp(V^s(y_t, a_t, B_t, \delta_{ht}))}{\exp(V^d(y_t, a_t, B_t, \delta_{ht}))}$$

$$\begin{aligned} \tilde{P}^d(y_t, a_t, B_t, \delta_{ht}) &= \frac{1}{\frac{1}{P^d(y_t, a_t, B_t, \delta_{ht})} + \frac{\exp(V^c(y_t, a_t, B_t, \delta_{ht}))(\exp(PE(t)) - 1)}{\exp(V^d(y_t, a_t, B_t, \delta_{ht}))}} \\ &= \frac{P^d(y_t, a_t, B_t, \delta_{ht})}{1 + \frac{\exp(V^c(y_t, a_t, B_t, \delta_{ht}))(\exp(PE(t)) - 1)}{\exp(V^d(y_t, a_t, B_t, \delta_{ht}))} P^d(y_t, a_t, B_t, \delta_{ht})} \end{aligned}$$

Note that $\tilde{P}^d(y_t, a_t, B_t, \delta_{ht}) > P^d(y_t, a_t, B_t, \delta_{ht})$ if

$$\frac{\exp(V^c(y_t, a_t, B_t, \delta_{ht}))(\exp(PE(t)) - 1)}{\exp(V^d(y_t, a_t, B_t, \delta_{ht}))} P^d(y_t, a_t, B_t, \delta_{ht}) < 0$$

As $\exp(\cdot) \geq 0$ and $P^d(y_t, a_t, B_t, \delta_{ht}) \geq 0$, this is possible only if $(\exp(PE(t)) - 1) < 0$, which occurs if $PE(t) < 0$. As is intuitive then, if the net externality from repayment is negative (due to a greater impact on asset buyers rather than sellers), the Planner chooses to default more than the private agent does.

Pecuniary externalities associated with default choice, expressed using discrete choice *probabilities*, will be a continuous function of the price p . The gap/wedge between the aggregate excess housing demand function in the competitive and constrained efficient allocations stems from these pecuniary externalities, and as the housing demand and supply functions are continuous in p , so is the wedge. Hence, if pecuniary externalities evaluated at the competitive equilibrium price p^* are small, then the neighborhood of p^* , since PE is continuous in p , the pecuniary externalities should remain small and thus the social planner's excess demand function should be close to zero. Thus, the planner's equilibrium price \hat{p} should be close to p^* and therefore $PE(\hat{p}) \approx PE(p^*)$.

Going from discrete choice probabilities to excess demand probability functions

The excess demand function is defined as:

$$z(p_t) = \sum_{y \in Y} \int_{a \in A} \omega(s_t) d\mu^R(s_t) - \sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} \left(\sigma(s_t) + \delta(s_t) \right) d\mu^O(s_t)$$

One can similarly define an excess demand probability function as:

$$z^P(p_t) = \sum_{y \in Y} \int_{a \in A} P^b(s_t) d\mu^R(s_t) - \sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} \left(P^s(s_t) + P^d(s_t) \right) d\mu^O(s_t)$$

Unlike the excess demand function, the excess demand probability function²⁷ is generally continuous in prices in the numerical solution. More importantly, one can define the Planner's excess demand function evaluated using the choice probabilities derived above as follows:

$$z_{sp}^P(p_t) = \sum_{y \in Y} \int_{a \in A} P^b(s_t) d\mu^R(s_t) - \sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} \left(\tilde{P}^s(s_t) + \tilde{P}^d(s_t) \right) d\mu^O(s_t)$$

The demand side of the above expression is unchanged, as the Planner does not intervene on that margin. However, note that if $\tilde{P}^s(s_t) \leq P^s(s_t)$ and $\tilde{P}^d(s_t) \leq P^d(s_t)$, then $z_{sp}^P(p_t) \geq z^P(p_t)$. In particular, this implies that at the competitive equilibrium price p_t^* , $z_{sp}^P(p_t^*) \geq 0$ and hence the equilibrium price under the Planner, $\tilde{p}_t \geq p_t^*$.

Further, from the Planner's choice probability expressions, the deviation $|\tilde{P}^j(s_t) - P^j(s_t)|$ is proportional to $PE(t)$. Hence, if $PE(t)$ is small, $\tilde{P}^j(s_t) \approx P^j(s_t)$.

1.9.2.4 Approximating the Lagrange multiplier on the borrowing constraint

An important component of the expression for collateral externalities in equation (1.19) is the Lagrange multiplier, η . I approximate the Lagrange multiplier by taking the difference between the marginal utility of consumption for a buyer, $u_c(y + aR - a' -$

²⁷I find that using the excess demand probability function to determine equilibrium prices yields almost equivalent results.

$\nu ph, \chi h$), and the numerical derivative w.r.t B of the expected owner value function, equation (1.19), the latter being evaluated in the neighborhood of the borrowing limit.

Although envelope conditions do not apply at the boundary of the choice set, i.e. the borrowing limit, I approximate the first order condition for borrowing choice (which can be derived by constructing suitable differentiable support functions for $q(y, a', b')$ and $V^o(y, a, b, \delta_h)$ in a manner similar to Clausen and Strub [2020]:

$$u_c(y + aR - a' + q(y, a', B')B' - ph, \chi h) + \frac{\beta EV_{B'}^o(y', a', B', \delta'_h)}{(q_{B'}B' + q)} = \eta(y, a) \quad (1.25)$$

I then approximate the expression above in a left neighborhood of the borrowing limit.

1.9.3 Updating the renter, owner and flagged agents' distributions

Given the initial distributions of owners, renters and flagged agents (μ_0^O , μ_0^R and μ_0^{aut} respectively)²⁸, the policy functions $\{a'(s), b'(s), \omega(s), \sigma(s), \delta(s)\}$ and the transition matrix for the Markov endowment process Π , this section describes how to update distributions $\{\mu^O, \mu^R, \mu^{aut}\}$ in a given period to new distributions denoted by $\{T\mu^O, T\mu^R, T\mu^{aut}\}$ in the next period, where T is the updating operator. This approach adapts the procedure for updating distributions in a standard heterogeneous agent model as described in Heer and Maussner [2009] to a housing model with defaulters, renters and owners (see also Corbae and Quintin [2015]).

Let the set of savings choices be A , the set of loan balances be B , the grid of endowments be Y and the set of depreciation shocks be Δ_h .

The updating process for the distributions is:

- Renter distribution (μ^R):

$$T\mu^R(y', a') = \sum_{y \in Y} \int_{a \in A} 1_{\{a'(y, a) = a'\}} * (1 - \omega(y, a)) * \Pi(y, y') * d\mu^R(y, a)$$

²⁸It is assumed in the quantitative model that all agents are renters in the beginning, and hence there is no mass of flagged defaulters.

$$\begin{aligned}
 & + \sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} 1_{\{a'(y, a, b, \delta_h) = a'\}} * \left(\sigma(y, a, b, \delta_h) \right) * \Pi(y, y') * d\mu^O(y, a, b, \delta_h) \\
 & + \sum_{y \in Y} \int_{a \in A} 1_{\{a'(y, a) = a'\}} * \theta * \Pi(y, y') * d\mu^{aut}(y, a)
 \end{aligned} \tag{1.26}$$

- Owner distribution (μ^O) :

$$\begin{aligned}
 T\mu^O(y', a', b', \delta'_h) = & \sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} 1_{\{a'(y, a, b, \delta_h) = a'\}} * 1_{\{\delta_m b = b'\}} * \left(1 - \sigma(y, a, b, \delta_h) \right) * \left(-\delta(y, a, b, \delta_h) \right) * \phi(\delta'_h) * \Pi(y, y') * d\mu^O(y, a, b, \delta_h) \\
 & + \sum_{y \in Y} \int_{a \in A} 1_{\{a'(y, a) = a'\}} * 1_{\{b'(y, a) = b'\}} * \omega(y, a) * \Pi(y, y') * d\mu^R(y, a)
 \end{aligned} \tag{1.27}$$

- Flagged agents' distribution (μ^{aut}) :

$$\begin{aligned}
 T\mu^{aut}(y', a') = & \sum_{y \in Y} \int_{a \in A} 1_{\{a'(y, a) = a'\}} * (1 - \theta) * \Pi(y, y') * d\mu^{aut}(y, a) \\
 & + \sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} 1_{\{a'(y, a, b, \delta_h) = a'\}} * \delta(y, a, b, \delta_h) * \Pi(y, y') * d\mu^O(y, a, b, \delta_h)
 \end{aligned} \tag{1.28}$$

Here, $1_{\{a'(y, a) = a'\}}$ is an indicator for whether the savings policy function for an agent with state (y, a) yields saving level a' . Similarly, $1_{\{b'(y, a) = b'\}}$ is an indicator for whether the loan balance choice for a renter with state (y, a) yields loan balance b' . Given the geometric mortgage decay parameter δ_m , $1_{\{\delta_m b = b'\}}$ is an indicator for whether an owner with balance b moves to balance b' in the next period.

In order to compute the stationary distributions $\{\mu^O, \mu^R, \mu^{aut}\}$, update the initial distributions $\{\mu_0^O, \mu_0^R, \mu_0^{aut}\}$ using the equations above.

1.9.4 An alternative formulation of the Social Welfare Function

The transition operator for distribution μ^j , $j = \{O, R, aut\}$, is given by $\mu'^j = T(\mu^j, \mu^{-j}, Q^j(s^j, s'^j))$, where $Q(., ., .)$ is the transition matrix from state s^j to s'^j for an individual agent and $T(., ., .)$ is the updating operator. Below, I do not use the superscript j to differentiate between the state vector and state space S^j for owners and renters, to avoid further cumbersome notation. I also abuse notation regarding the state spaces $\{Y, A, B, \Delta_h\}$ in order to economise on space. In particular, aggregating over the state space for current owners (for e.g.) is represented by $\sum_{(y,a,b,\delta_h) \in Y \times A \times B \times \Delta_h}$, rather than the accurate $\sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B}$. The consumption, saving and loan choices are policy functions, as the use of the respective value functions indicates.

In the following, I consider an alternative formulation of the SWF that the constrained Planner maximizes. I use the equations of motion for the distributions from appendix 1.9.3 to derive this expression.

Aggregate welfare among owners, renters and flagged agents is:

$$\begin{aligned} \mathcal{W}^{\mathcal{OR}} = & \sum_{(y,a) \in Y \times A} \left(u(c, h) + \sum_{s' \in S'} \beta [\omega(y, a) * V^o(y', a', b', \delta'_h) * Q^o(s, s') \right. \\ & \left. + (1 - \omega(y, a)) * V^r(y', a') * Q^r(s, s')] \right) * d\mu^R(y, a) \\ & + \left(\sum_{(y,a) \in Y \times A} u(c, h) + \sum_{s' \in S'} \beta [\theta * V^r(y', a') * Q^r(s, s') + (1 - \theta) * V^{aut}(y', a') * Q^{aut}(s, s')] \right) * d\mu^{aut}(y, a) \\ & + \sum_{(y,a,b,\delta_h) \in Y \times A \times B \times \Delta_h} \left(u(c, \chi h) + \sum_{s' \in S'} \beta [(1 - \delta) * \{ (1 - \sigma) * V^o(y', a', b', \delta'_h) * Q^o(s, s') + \sigma * V^r(y', a') * Q^r(s, s') \} \right. \\ & \left. + \delta * V^{aut}(y', a') * Q^r(s, s')] \right) * d\mu^O(y, a, b, \delta_h) \end{aligned} \quad (1.29)$$

This notation is awkward, as it subsumes the future distributions of owners and renters, (μ'^O, μ'^R) , into the products of the optimal buying/selling/defaulting choices

with the *current* densities for owners and renters. It is easier to demonstrate the externality wedge using the expression in equation (1.29) rather than the recursive one in equation (1.15).

Further, as discussed above, the Planner also cares about the amount recovered by lenders during a foreclosure:

$$\sum_{(y,a,b,\delta_h) \in Y \times A \times B \times \Delta_h} \left\{ \delta(y, a, b, \delta_h) * \min\{\kappa p h, b\} + \left(1 - \delta(y, a, b, \delta_h)\right) b \right\} * d\mu^O(y, a, b, \delta_h) \quad (1.30)$$

Thus, the total welfare function for the economy based on the transition function operators is obtained by adding up equations (1.29) and (1.30):

$$\begin{aligned} \mathcal{W} = & \underbrace{\sum_{(y,a) \in Y \times A} \left(u(c, h) + \beta \sum_{s' \in S'} \left[\omega(y, a) * V^o(y', a', b', \delta'_h) * Q^o(s, s') \right. \right.} \\ & \left. \left. + \left(1 - \omega(y, a)\right) * V^r(y', a') * Q^r(s, s') \right] \right) * d\mu^R(y, a)}_{\text{Renter's welfare}} \\ & \underbrace{\left. + \left(\theta * V^r(y', a') * Q^r(s, s') + (1 - \theta) * V^{aut}(y', a') * Q^{aut}(s, s') \right) \right) * d\mu^{aut}(y, a)}_{\text{Flagged agents' welfare}} \\ & \underbrace{\sum_{(y,a,b,\delta_h) \in Y \times A \times B \times \Delta_h} \left(u(c, \chi h) + \beta \left[(1 - \delta) * \left\{ (1 - \sigma) * V^o(y', a', b', \delta'_h) * Q^o(s, s') + \sigma * V^r(y', a') * Q^r(s, s') \right\} \right. \right.} \\ & \left. \left. + \delta * V^{aut}(y', a') * Q^{aut}(s, s') \right] \right) * d\mu^O(y, a, b, \delta_h)}_{\text{Owners' welfare}} \\ & \underbrace{\left. + \delta * V^{aut}(y', a') * Q^{aut}(s, s') \right) * d\mu^O(y, a, b, \delta_h)}_{\text{Owners' welfare (contd.)}} \\ & + \underbrace{\sum_{(y,a,b,\delta_h) \in Y \times A \times B \times \Delta_h} \left\{ \delta * \min\{\kappa p h, b\} + \left(1 - \delta(y, a, b, \delta_h)\right) b \right\} * d\mu^O(y, a, b, \delta_h)}_{\text{Lenders' welfare}} \end{aligned} \quad (1.31)$$

1.9.5 Planner's default choice and envelope conditions

This appendix features a proof of proposition 1 about the default choice of a Planner, provides a complete expression for the total pecuniary externality and its distributive

component, and derives envelope conditions.

1.9.6 Default choice

Proposition 1: The Planner chooses default $(\delta(y, a, b, \delta_h) = 1)$ for an owner if:

$$V^d(y, a, b) - \max \left\{ V^s(y, a, b, \delta_h), V^c(y, a, b, \delta_h) \right\} \\ + \underbrace{\min\{\kappa ph, b\} - b}_{\text{Lender loss from foreclosure}} + \underbrace{\text{PE}}_{\text{pecuniary externality}} > 0 \quad (1.32)$$

Proof. The approach here is to use a perturbation argument in the spirit of [Davila et al. \[2012\]](#). Throughout the proof, I use the expression for the SWF in equation (1.31). Hence, the constrained Planner's problem is to maximize equation (1.31) w.r.t $\delta(y, a, b, \delta_h)$ subject to other decentralized choices and equilibrium loan and house price relationships.

Let the Planner's default choice be $\delta(y, a, b, \delta_h)$. Consider a perturbation where the positive mass $d\mu^O(y, a, b, \delta_h)$ of owners with state (y, a, b, δ_h) for whom the unique optimal choice is to default are now switched to non-defaulting status, i.e. they switch from $\delta(y, a, b, \delta_h) = 1$ to $\delta(y, a, b, \delta_h) = 0$. This reduces housing supply by an amount $\Delta H = d\mu^O(y, a, b, \delta_h)$, and I denote the resulting price change by $\Delta p \geq 0$. Finally, let the change in felicity of an agent due to a price change be denoted by $\Delta_p u(c, h; p)$.

The resulting change in social welfare from the perturbation, denoted by ΔW , is:

$$\Delta W = \left(\max \left\{ V^c(y, a, b, \delta_h), V^s(y, a, b, \delta_h) \right\} - V^d(y, a, b) \right) * d\mu^O(y, a, b, \delta_h) \\ - \min\{\kappa ph - b, 0\} * d\mu^O(y, a, b, \delta_h) \\ + \sum_{y' \in Y} \int_A \omega(y', a') \left(\eta(y', a') * (1 - \iota) * \Delta p + \Delta_p u(c, \chi h; p) \right) * d\mu^R(y', a') \\ + \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} \left(\delta(y', a', b', \delta'_h) * \chi_{\{\kappa ph < b'\}} * \Delta p \right) * d\mu^O(y', a', b', \delta'_h)$$

$$\begin{aligned}
 & + \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} \left((1 - \delta(y', a', b', \delta'_h)) * \sigma(y', a', b', \delta'_h) * \Delta_p u(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h) \\
 & + \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} \left((1 - \delta(y', a', b', \delta'_h)) * (1 - \sigma(y', a', b', \delta'_h)) * \Delta_p u(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h)
 \end{aligned}$$

Using the first order approximation $u'(c)\Delta c \approx \Delta u$, consider for example the change in utility of sellers due to the price change. Seller consumption is $c = y + aR + p*(1 - \kappa_h - \delta_h) - b - a' - \rho$, so $\Delta c = (1 - \kappa_h - \delta_h)\Delta p$. Hence, $\Delta_p u(c, h; p) = u_c(c, h; p) * (1 - \kappa_h - \delta_h) * \Delta p = -u_c(c, h; p) * (1 - \kappa_h - \delta_h) * \frac{\Delta p}{\Delta H} * d\mu^O(y, a, b, \delta_h)$. Similarly, for owners who repay, the change in utility is $\Delta_p u(c, \chi h; p) = u_c(c, \chi h; p) * (\delta_h) * \frac{\Delta p}{\Delta H} * d\mu^O(y, a, b, \delta_h)$; and for buyers, the change in utility is $\Delta_p u(c, \chi h; p) = u_c(c, \chi h; p) * (1 + \kappa_b) * \frac{\Delta p}{\Delta H} * d\mu^O(y, a, b, \delta_h)$. Thus, the above expression becomes:

$$\begin{aligned}
 \Delta W \approx & \left(\max \{V^c(y, a, b, \delta_h), V^s(y, a, b, \delta_h)\} - V^d(y, a, b) \right) * d\mu^O(y, a, b, \delta_h) \\
 & - \min\{\kappa p h - b, 0\} * d\mu^O(y, a, b, \delta_h) \\
 & - \left\{ \sum_{y' \in Y} \int_A \omega(y', a') \left(\eta(y', a') * (1 - \iota) - u_c(c, \chi h; p) * (1 + \kappa_b) \right) * d\mu^R(y', a') \right\} * \frac{\Delta p}{\Delta H} * d\mu^O(y, a, b, \delta_h) \\
 & - \left\{ \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} \kappa(\delta(y', a', b', \delta'_h) * \chi_{\{\kappa p h < b'\}}) * d\mu^O(y', a', b', \delta'_h) \right\} * \frac{\Delta p}{\Delta H} * d\mu^O(y, a, b, \delta_h) \\
 & - \left\{ \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} (1 - \kappa_h - \delta'_h) * \left((1 - \delta(y', a', b', \delta'_h)) * \sigma(y', a', b', \delta'_h) * u_c(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h) \right. \\
 & \left. + \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} (\delta'_h) * \left((1 - \delta(y', a', b', \delta'_h)) * (1 - \sigma(y', a', b', \delta'_h)) * u_c(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h) \right\} \\
 & * \frac{\Delta p}{\Delta H} * d\mu^O(y, a, b, \delta_h)
 \end{aligned}$$

Rewriting the expression above, one obtains:

$$\begin{aligned}
 \Delta W \approx & \left\{ \left(\max \{V^c(y, a, b, \delta_h), V^s(y, a, b, \delta_h)\} - V^d(y, a, b) \right) \right. \\
 & \left. - \min\{\kappa p h - b, 0\} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & - \sum_{y' \in Y} \int_A \omega(y', a') \left(\eta(y', a') * (1 - \iota) - u_c(c, \chi h; p) * (1 + \kappa_b) \right) * d\mu^R(y', a') * \frac{\Delta p}{\Delta H} \\
 & - \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} \kappa(\delta(y', a', b', \delta'_h) * \chi_{\{\kappa p h < b'\}}) * d\mu^O(y', a', b', \delta'_h) * \frac{\Delta p}{\Delta H} \\
 & - \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} (1 - \kappa_h - \delta'_h) * \left((1 - \delta(y', a', b', \delta'_h)) * \sigma(y', a', b', \delta'_h) * u_c(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h) * \frac{\Delta p}{\Delta H} \\
 & + \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} (\delta'_h) * \left((1 - \delta(y', a', b', \delta'_h)) * (1 - \sigma(y', a', b', \delta'_h)) * u_c(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h) * \frac{\Delta p}{\Delta H} \Big\} \\
 & * d\mu^O(y, a, b, \delta_h)
 \end{aligned}$$

As the Planner's optimal choice assigns all agents with state (y, a, b, δ_h) to default, the perturbation should not increase welfare. Further, if there were no change in welfare, then default and the preferred option between sale or repayment would both be optimal to the Planner for agents with state (y, a, b, δ_h) . As we have assumed that default is the unique optimal choice for the Planner for agents with state (y, a, b, δ_h) , the perturbation must yield strictly lower welfare. Hence, $\Delta W < 0$, and since $d\mu^O(y, a, b, \delta_h) > 0$, the expression in braces must be negative. Thus, the Planner chooses default if the following condition holds:

$$\begin{aligned}
 & \left(V^d(y, a, b) - \max \{ V^c(y, a, b, \delta_h), V^s(y, a, b, \delta_h) \} \right) \\
 & + \min \{ \kappa p h - b, 0 \} \\
 & + \sum_{y' \in Y} \int_A \omega(y', a') \left(\eta(y', a') * (1 - \iota) - u_c(c, \chi h; p) * (1 + \kappa_b) \right) * d\mu^R(y', a') * \frac{\Delta p}{\Delta H} \\
 & + \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} \kappa(\delta(y', a', b', \delta'_h) * \chi_{\{\kappa p h < b'\}}) * d\mu^O(y', a', b', \delta'_h) * \frac{\Delta p}{\Delta H} \\
 & + \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} (1 - \kappa_h - \delta'_h) * \left((1 - \delta(y', a', b', \delta'_h)) * \sigma(y', a', b', \delta'_h) * u_c(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h) * \frac{\Delta p}{\Delta H} \\
 & - \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} (\delta'_h) * \left((1 - \delta(y', a', b', \delta'_h)) * (1 - \sigma(y', a', b', \delta'_h)) * u_c(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h) * \frac{\Delta p}{\Delta H} > 0
 \end{aligned}$$

□

1.9.6.1 The Pecuniary externality

Based on the above derivation, I express the total pecuniary externality associated with default choice for marginal changes in supply (with $\frac{\Delta p}{\Delta H} \leq 0$) as:

$$\begin{aligned}
 \text{PE} = & \sum_{y' \in Y} \int_A \omega(y', a') \left(\eta(y', a') * (1 - \iota) u_c(c, \chi h; p) * (1 + \kappa_b) \right) * d\mu^R(y', a') * \frac{\Delta p}{\Delta H} \\
 & \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} \kappa(\delta(y', a', b', \delta'_h) * \chi_{\{\kappa p h < b'\}}) * d\mu^O(y', a', b', \delta'_h) * \frac{\Delta p}{\Delta H} \\
 & + \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} (1 - \kappa_h - \delta'_h) * \left((1 - \delta(y', a', b', \delta'_h)) * \sigma(y', a', b', \delta'_h) * u_c(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h) * \frac{\Delta p}{\Delta H} \\
 & - \sum_{y' \in Y} \sum_{\delta'_h \in \Delta_h} \int_{A \times B} (\delta'_h) * \left((1 - \delta(y', a', b', \delta'_h)) * (1 - \sigma(y', a', b', \delta'_h)) * u_c(c, h; p) \right) * d\mu^O(y', a', b', \delta'_h) * \frac{\Delta p}{\Delta H}
 \end{aligned}$$

1.9.6.2 The distributive externality expression

$$\begin{aligned}
 \text{Distributive externality} = & \underbrace{\left[\sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} u_c(c, h) * \sigma(y, a, b, \delta_h) * (1 - \kappa_h - \delta_h) * d\mu^O(y, a, b, \delta_h) \right]}_{\text{Impact on sellers}} \\
 & - \underbrace{\sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} u_c(c, h) * \left((1 - \sigma(y, a, b, \delta_h)) (1 - \delta(y, a, b, \delta_h)) \right) * \delta_h * d\mu^O(y, a, b, \delta_h)}_{\text{Impact on owners}} \\
 & - \underbrace{\sum_{y \in Y} \int_{a \in A} u_c(c, h) * \omega(y, a) * (1 + \kappa_b) * d\mu^R(y, a)}_{\text{Impact on homebuyers}} \\
 & + \underbrace{\sum_{y \in Y} \sum_{\delta_h \in \Delta_h} \int_{a \in A} \int_{b \in B} \kappa * 1_{\{\kappa p h < b\}} * \delta(y, a, b, \delta_h) * d\mu^O(y, a, b, \delta_h)}_{\text{Impact on lenders}} \Bigg] * \frac{\Delta p}{\Delta H} \quad (1.33)
 \end{aligned}$$

Chapter 2

Pecuniary externalities and constrained inefficiency in an intermediated directed search model of the housing market

2.1 Introduction

The quantitative literature on housing markets in a heterogeneous agents environment has increasingly studied the 2006-2011 housing bust using models that incorporate frictions in house purchase and sale (see e.g. [Hedlund \[2016a,b\]](#), [Garriga and Hedlund \[2020\]](#)). This was intended to capture selling delays and the price and consumption declines during the Great Recession, or to better understand the interaction of such frictions with other factors behind the housing crash (such as tighter credit or income shocks). One question that has hitherto not been explored in the literature is about the presence and corrective policy implications of pecuniary externalities associated with frictional house trades. In an environment with financial frictions, one would generally expect pecuniary externality-driven inefficiencies arising from sale or default choices that would motivate policy intervention. Indeed, the first chapter of this dissertation verifies this assertion in a frictionless or *Walrasian* housing market.

This article confirms that the above intuition also holds true in an environment with frictional house trades. It employs an intermediated directed search, incomplete

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markets model of the housing market, following the aforementioned literature. It shows that the decision to sell a house rather than repay mortgage debt is *constrained inefficient*. The analysis is *normative* and *ex post*: A social planner chooses sale on behalf of owners in order to maximize a social welfare objective subject to market incompleteness (due to uninsurable idiosyncratic risk), financial frictions *and* trading frictions. Hence, this is a constrained planner's problem in the tradition of [Stiglitz \[1982\]](#) and [Geanakoplos and Polemarchakis \[1986\]](#). The planner also takes the other decentralized choices made by agents as given when choosing whether to sell a house, i.e. the planner only intervenes along the margin of sale choice. For simplicity, I assume that the planner's intervention to affect sale choice is not anticipated by agents in the economy.

Constrained inefficiency, expressed as the wedge between the planner's choice of sale and private sale choice, arises due to pecuniary externalities operating through a price index. The framework incorporating directed search and heterogeneity implies that sellers choose market tightness, i.e. the broker-seller ratio, in housing *submarkets* differentiated by their characteristics such as income or asset holdings. Each submarket is characterized by a different list (posted) price.

The key assumption that allows pecuniary externalities to operate is that all housing trades are intermediated by brokers. The quasi-Walrasian price index that clears the broker-intermediated housing market affects agents' choices in each individual submarket and thereby links the different submarkets together. This is the channel through which pecuniary externalities¹ associated with sale choices operate. Hence, the assumption of broker-intermediated trades that the quantitative literature makes for reasons of tractability is important in driving the pecuniary externality-based inefficiency results.

The overall sign of the pecuniary externality depends on whether sellers or buyers are more constrained in the aggregate. Intuitively, if sellers tend to be more constrained, then house sales are inefficiently high as individual sellers do not internalize that the overall losses to other sellers from receiving lower prices outweigh the gains to buyers from having to pay a lower price.

When the model is enriched to consider defaultable debt, I find that default and

¹The pecuniary externality studied here is distributive in nature and arises due to imperfect risk sharing. See [Dávila and Korinek \[2018\]](#) for a characterization of pecuniary externalities in models with financial frictions.

foreclosure introduces an *additional* inefficiency associated with realized (*ex post*) lender foreclosure losses or deadweight costs.

Directed search models are generally constrained efficient with regard to market tightness choice (the buyer-seller ratio): the choice of market tightness by a planner subject to the same search and matching frictions as private agents coincides with the decentralized tightness choice. Directed search models allow agents to trade off their returns from trade (posted prices) with the probability of being matched. This endogenously satisfies the Hosios condition for efficiency in search and matching models (see e.g. [Rogerson et al. \[2005\]](#), [Wright et al. \[2021\]](#)). Results on inefficiency in directed search models often hinge on informational frictions, e.g. private information about match-specific productivity for workers ([Faig and Jerez \[2005\]](#), [Guerrieri \[2008\]](#)), or product quality for sellers ([Guerrieri et al. \[2010\]](#), [Guerrieri and Shimer \[2014\]](#)).

I also examine whether individual market tightness choices (the ratio of brokers to sellers in this framework) are constrained efficient. I show that pecuniary externalities introduce a wedge between private and constrained efficient market tightness choice. For example, if sellers are more constrained than buyers, then the pecuniary externality would lead to a planner choosing lower market tightness and thereby reducing the probability of a successful sale to keep the price index high and thereby benefit other sellers in the economy. One could alternatively express this inefficiency in terms of list prices: private sellers set list prices that are too low relative to the efficient level.

To the best of my knowledge, the constrained inefficiency of tightness choice (or list prices) due to pecuniary externalities has not been derived before in the directed search literature. Trade intermediation by brokers is again at the heart of this result. Hence, this chapter contributes to the aforementioned literature by highlighting the role of pecuniary externalities in driving inefficiencies associated with both the decision to sell as well as the sale list price in a model with frictional house trades.

2.1.1 Related literature

Pecuniary externalities and inefficiency arising therefrom have been discussed extensively in various environments (see [Dávila and Korinek \[2018\]](#) and the references therein). The classification of pecuniary externalities as being distributive or collateral in nature was made by [Dávila and Korinek \[2018\]](#). Though the emphasis in this literature has been on intervention *ex ante*, there is also a smaller literature that

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evaluates the use and implications of *ex post* intervention owing to the same source of inefficiency (e.g. [Jeanne and Korinek \[2020\]](#)). Pecuniary externalities in a standard incomplete markets model were first investigated by [Davila et al. \[2012\]](#).

The first chapter of this dissertation derives the inefficiencies associated with mortgage default choices in a standard incomplete markets housing model where the housing market is Walrasian (frictionless). The key difference between that chapter and the present chapter is the treatment of the housing market, which here explicitly incorporates search frictions using the directed search approach. In addition, I make certain simplifying assumptions in this chapter as my objective here is to demonstrate theoretically the robustness of the inefficiency results derived previously in a Walrasian environment. For instance, I study one-period mortgage debt contracts for simplicity here, as opposed to the treatment in the first chapter that models mortgages as long-term debt contracts. Also, the baseline results in this paper describe inefficiencies arising due to house sales. Finally, this chapter does not consider collateral externalities associated with collateral constraints for home buyers.

Seminal papers introducing the directed search approach in the macro-labor literature are [Moen \[1997\]](#) and [Acemoglu and Shimer \[1999\]](#), who also discuss efficiency in relation to the canonical labor search and matching literature discussed in, e.g. [Pissarides \[2000\]](#). [Wright et al. \[2021\]](#) is a recent survey that discusses efficiency and other applications of the directed search framework.

Macroeconomic models with housing are discussed in, e.g., [Davis and Van Nieuwerburgh \[2015\]](#) and [Piazzesi and Schneider \[2016\]](#). A branch of literature using search models to study liquidity in the housing market includes [Wheaton \[1990\]](#), [Ngai and Tenreyro \[2014\]](#), [Head et al. \[2014\]](#). These models do not either allow for or consider how credit or net worth affects housing. [Guren and McQuade \[2020\]](#) use a search model to study the feedback between foreclosures and house prices, but also does not allow for borrowing or saving. Recent quantitative models that use a heterogeneous agents directed search model include [Hedlund \[2016a,b\]](#), [Garriga and Hedlund \[2020\]](#), [Jerez et al. \[2020\]](#). These articles discuss sorting of market tightness choices by agents' financial state variables, which I consider as well. However, they do not discuss pecuniary externalities or constrained inefficiency, which is the primary focus of this paper.

2.1.2 Outline of paper

Section 2.2 describes the model and the stationary equilibrium. Section 2.3 examines how the market tightness choices of potential sellers vary with their risk-free asset holdings. Section 2.4 describes the pecuniary externality based inefficiency associated with the decision to sell. Section 2.5 discusses some extensions concerning frictional house purchases, the default option and the inefficiency associated with tightness choice. Section 2.6 concludes. Supplementary material is contained in two appendices. An outline of the material in the appendices can be found at the beginning of appendix 2.7.1.

2.2 Model

The model is an infinite horizon incomplete markets model with housing. Heterogeneity arises *ex post* due to the saving and borrowing choices of agents facing uninsurable idiosyncratic risk, in addition to the tenancy/ownership choice. Trading on the supply side of the housing market is frictional, whereas for simplicity homebuyers are assumed to trade frictionlessly. All trades are intermediated by real estate brokers.

There are four types of agents in the economy: (i) *renters*, who choose whether to own a house by taking out a mortgage or remain as tenants; (ii) *owners*, who decide whether to sell their house through a directed search mechanism or repay their mortgage; (iii) risk-neutral *financial intermediaries/lenders* who lend to homebuyers; and (iv) risk-neutral *brokers*, who purchase houses from owners in a competitive search process and in turn sell houses frictionlessly to buyers.

2.2.1 Environment

Time is discrete, continues forever and is indexed by $t = 0, 1, 2, \dots$. There is a continuum of agents who receive an endowment y drawn independently according to a Markov process with values in finite set Y . The probability that an endowment transitions from current level of y to y' is given by the transition matrix $\Pi(y'|y)$. In the following, I use primes to denote variables in the next period.

Agents have period utility functions $u(c, \chi h)$, hence they receive benefits from consumption and housing services if they are owners (the housing preference parameter

$\chi > 1$ for owners). For simplicity, it is assumed that rental housing yields housing services one-for-one with house size, i.e. $\chi = 1$. Utility is separable between consumption and housing services. All agents discount the future using discount factor β .

2.2.2 Housing market

Housing (owner-occupied and rental) is of a single size ($h = 1$), and agents can only own one house at a time. There is a fixed total housing stock and no construction sector. The owner-occupied and rental sectors of the housing market are segmented, so there is no convertibility between owner-occupied and rental housing space, which would otherwise tie down a relationship between house prices and rents (see e.g. [Kaplan et al. \[2019\]](#), [Greenwald and Guren \[2019\]](#)). This also implies that the homeownership rate is fixed in the economy. Rents are fixed and are assumed to be earned by absentee landlords. Endogenous rents would introduce a channel of pecuniary externalities that would arise through rents, which I do not consider in this paper.

Heterogeneous home sellers trade their houses in a frictional decentralized market, following [Garriga and Hedlund \[2020\]](#). The trading mechanism employs the directed search approach, wherein sellers post their prices and their trade counterparts direct their search accordingly. As is typical in directed search models, sellers face a tradeoff between the price they post and the probability of sale.

I follow the directed search housing literature (e.g. [Hedlund \[2016b\]](#), [Garriga and Hedlund \[2020\]](#)) and assume that all housing trades are intermediated by real estate brokers in order to account for two-sided heterogeneity on the side of buyers and sellers. As these papers discuss, in the absence of brokers, matching between heterogeneous sellers and buyers requires each party to forecast the dynamics of the entire distribution of income, assets and debt in order to calculate their trading probability in each submarket. Introducing brokers breaks this down to a one-sided heterogeneous matching problem, which considerably simplifies the analysis.

Hence, brokers buy houses from sellers in a frictional market, and can trade with other brokers and with buyers in a frictionless market at price index p that clears the broker-intermediated housing market in equilibrium. Introducing frictional trading on the demand side adds complexity without adding to the basic result of the model regarding pecuniary externalities associated with house sales, hence I make the

Walrasian trading assumption on the demand side. Section 2.5.1 describes how the main result would be altered if home purchases were also frictional.

Search in the housing market

The probability $\eta(\theta)$ that a seller matches with a broker in any given *submarket* depends on the ratio of brokers to sellers in that submarket (θ), i.e. the *market tightness*. A submarket is characterized by market tightness θ and terms of trade $p(\theta)$. I assume that $\eta'(\theta) > 0$, $\eta''(\theta) < 0$ and η is \mathbb{C}^2 , with $\eta(0) = 0$ and $\lim_{\theta \rightarrow \infty} \eta(\theta) = 1$. The corresponding probability that brokers match with a seller in a given submarket with tightness θ is then $\alpha(\theta) = \frac{\eta(\theta)}{\theta}$. Further, $\alpha(\theta)$ is strictly decreasing and C^2 , with $\lim_{\theta \rightarrow \infty} \alpha(\theta) = 0$ and $\lim_{\theta \rightarrow 0} \alpha(\theta) = 1$. The elasticity $\epsilon(\theta) \equiv \frac{\eta'(\theta)\theta}{\eta(\theta)}$ is assumed to be non-increasing.

Jerez et al. [2020] extend the matching functions η and α to domain $\Theta \equiv \mathbb{R}_+ \cup \{\theta_0\}$, with fictitious submarket $\theta_0 \in \mathbb{R}_-$ such that $\eta(\theta_0) = \alpha(\theta_0) = 0$. This fictitious submarket represents owners who choose not to sell but to repay their mortgage debt, hence it corresponds to non-participation in the frictional house sale market. As seen below, I consider the sale decision σ explicitly, implying that non-participation in the frictional house sale market $(1 - \sigma)$ is equivalent to choosing θ_0 and hence $\eta(\theta_0) = \alpha(\theta_0) = 0$.

Brokers face transaction costs κ_s and receive net revenue $(p - p(\theta))$ when they match with a seller. Free entry of brokers into each submarket and the zero-profit condition require:

$$\kappa_s \geq \alpha(\theta)(p - p(\theta)) \quad (2.1)$$

The combination of directed search and free entry of brokers implies that market tightness and hence sale probabilities depend only on the price index p and *not* on the distributions of owners and renters in the economy. This is referred to in the literature as *block recursivity* (see also Menzio and Shi [2010], Menzio et al. [2013], Wright et al. [2021] for a discussion in other contexts).

2.2.3 Financial markets

All households can save in a risk-free asset at rate R . Additionally, home buyers can finance their house purchases by borrowing.

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Mortgages are one-period debt contracts, and I assume that owners can also refinance their debt each period. These assumptions help make the analysis of the constrained planner's problem tractable. As I assume that mortgages are non-defaultable, the borrowing limit depends on the worst possible realization of income (denoted by \underline{y}). Thus, if a' represents saving (which is negative if an owner borrows), then the borrowing constraint faced by owners can be written as:

$$a' \geq -\underline{y} \tag{2.2}$$

Loan price in the absence of default is then simply R^{-1} .

2.2.4 Choices and value functions

The choices of different types of agents can be summarized as follows:

- Current owners can *continue* as owners and make their mortgage payment (with or without refinancing) or *sell* their house. Unsuccessful sales lead to continuation and repayment.
- Renters can *buy* a house and become owners, or continue to *rent*.

In the notation to follow, the short term constraint for tenants and sellers precludes borrowing, $a'(s) \geq 0$, i.e. the domain for saving choice is the set $a' \in \mathbb{R}_+$.

Current owner

As mentioned above, a current owner in each period chooses whether to continue with ownership by repaying, or sell his house ($\sigma = 1$). Upon successful sale, the agent loses possession of his house immediately and rents in that period. I denote the fixed rent by ρ below. If sale is unsuccessful, the owner has to repay the mortgage and remain an owner. The state variables for an owner in each period include his endowment y and asset position a . Let the vector of state variables be $s^o = (y, a)$. The expectations operator \mathbb{E} is defined over y' given s^o using transition probabilities Γ .

2.2.4.1 Continuation and matched seller value functions

I represent the value of continuing with a mortgage contract by $V^c(s_t^o)$. Then, the Bellman equation for continuing is:

$$V^c(s_t^o) = \max_{\{c, a'\}} u(c, \chi h) + \beta \mathbb{E} \max \left\{ V^c(s_{t+1}^o), V^s(s_{t+1}^o) \right\} \quad (2.3)$$

subject to

$$c + \frac{a'}{R} = y + a$$

and

$$a' \geq -\underline{y}$$

If a potential seller is *successfully matched* and sells his house, he keeps the proceeds from selling his house after paying off his loan (if $a < 0$). The Bellman equation for a matched seller is:

$$v^s(y, a) = \max_{\{c, a' \geq 0\}} u(c, h) + \beta \mathbb{E} V^r(y', a') \quad (2.4)$$

subject to

$$c + \frac{a'}{R} + \rho = y + a + p(\theta(s^o))$$

2.2.4.2 Potential seller value function

An owner with state s_t^o who chooses to sell need not be matched successfully, due to the frictional sale mechanism. His sale probability $\eta(s_t^o)$ depends on the market tightness choice, $\theta(s_t^o)$ and the list price chosen, $p(\theta(s_t^o)) \in \mathbb{R}_+$.

The potential seller then chooses market tightness $\theta(s_t^o)$ in order to maximize his expected payoff from choosing sale ($\sigma(s_t^o) = 1$), which I denote by $V^s(s_t^o)$:

$$V^s(s_t^o) = \max_{\theta(s_t^o) \in \Theta} \eta(\theta(s_t^o)) v^s(s_t^o) + \left(1 - \eta(\theta(s_t^o))\right) V^c(s_t^o) \quad (2.5)$$

subject to

$$p(\theta(s_t^o)) = p - \frac{\kappa_s}{\alpha(\theta(s_t^o))} \geq -a - y \quad (2.6)$$

The constraint in equation (2.6) is the zero profit condition for brokers. It is similar to other directed search models wherein the agent choosing market tightness has to ensure that his trading counterpart meets a certain reservation utility level². In this model, free entry of brokers into active submarkets (which is the subset of Θ that contains the solutions to the above problem, i.e. those submarkets that attract both sellers and brokers) implies that they must be guaranteed zero profits, hence the list price must satisfy equation (2.6) in active submarkets. Sellers therefore internalize the 'participation constraint' for brokers (the list price that yields brokers zero profits) when choosing submarket tightness.

Jerez [2014] shows that one could instead consider a problem wherein both sellers and brokers choose submarket tightnesses taking the price function $p(\theta)$ as given, which she refers to as the 'price taking approach'. Houses traded in different submarkets θ can be thought of as different commodities, and the price function $p : \Theta \rightarrow \mathbb{R}_+$ yields house prices in different submarkets. In equilibrium, sellers and brokers take $p(\theta)$ as given and have rational expectations about the tightness levels in active submarkets.

Equation (2.6) implies a tradeoff between the list price and trade probability, as $\alpha'(\theta) < 0$. This is standard in directed search models. Note that one could rewrite the problem for a potential seller as the choice of a list price, with the market tightness being based on expressing equation (2.6) in terms of θ .

The price function is bounded above by $p - \kappa_s$. I shall focus on when sellers choose a non-negative list price, i.e. their choice of θ yields a $p(\theta) \geq 0$.

If the constraint binds, θ solves the following equation:

$$\alpha(\theta) = \frac{\kappa_s}{p} \quad (2.7)$$

The owner value function for each state vector is defined as the upper envelope of the value functions associated with continuation and potential sale defined at the

²This is referred to in the directed search literature as the market utility approach.

same state vector:

$$V^o(s_t^o) = \max \left\{ V^c(s_t^o), V^s(s_t^o) \right\} \quad (2.8)$$

Renter

The renter can either purchase a home through a mortgage ($\omega = 1$), in which case he becomes an owner, or choose to remain a renter ($\omega = 0$), which is referred to as tenancy. The relevant state variables for a renter are his income y and asset level a , so a renter's state vector $s^r = (y, a)$.

A renter who chooses not to purchase a house remains a **tenant**, paying rent ρ , and has the value function:

$$V^t(s^r) = \max_{\{c, a' \geq 0\}} u(c, h) + \beta \mathbb{E} V^r(y', a') \quad (2.9)$$

subject to

$$c + \frac{a'}{R} + \rho = y + a$$

A renter who chooses to buy a house will do so by purchasing a mortgage. Given house value p and transaction cost that is proportional to the house price index, $\kappa_b p$, his initial asset choice would be a' .

It is assumed that a buyer enjoys homeownership utility premium in the period of purchase, i.e. he gets immediate possession of the house. Therefore, his value function would be:

$$V^b(s^r) = \max_{\{c, a'\}} u(c, \chi h) + \beta \mathbb{E} V^o(y', a') \quad (2.10)$$

subject to the borrowing constraint in equation (2.2) and the budget constraint,

$$c + \frac{a'}{R} = -p(1 + \kappa_b) + a + y$$

The renter value function for each state vector is defined as the upper envelope of the tenant's and homebuyer's value functions defined at the same state vector:

$$V^r(s^r) = \max \left\{ V^t(s^r), V^b(s^r) \right\} \quad (2.11)$$

2.2.5 Distributions of owners and renters

The distributions of owners (μ^o) and renters (μ^r) are defined over the relevant state space $Y \times \bar{A}$, where $\bar{A} = \{a : a \geq \underline{a}\}$. If $\underline{a} = 0$, $\bar{A} = \mathbb{R}_+$.

Given the initial distributions of owners and renters (μ_0^o, μ_0^r respectively), the policy functions $\{a'(s), \theta(s), \omega(s), \sigma(s)\}$ and the transition matrix for the Markov endowment process Π , this section describes the evolution of distributions from $\{\mu^o, \mu^r\}$ in a given period to new distributions denoted by $\{T\mu^o, T\mu^r\}$ in the next period, where T is the updating operator.

The updating process for the distributions is:

- Renter distribution (μ^r):

$$\begin{aligned} T\mu^r(y', a') &= \sum_{y \in Y} \int_{a \in \bar{A}} 1_{\{a'(y, a) = a'\}} * \left(1 - \omega(y, a)\right) * \Pi(y, y') * d\mu^r(y, a) \\ &+ \sum_{y \in Y} \int_{a \in \bar{A}} 1_{\{a'(y, a) = a'\}} * \left(\sigma(y, a) * \eta(\theta(y, a))\right) * \Pi(y, y') * d\mu^o(y, a) \end{aligned} \quad (2.12)$$

- Owner distribution (μ^o):

$$\begin{aligned} T\mu^o(y', a') &= \sum_{y \in Y} \int_{a \in \bar{A}} 1_{\{a'(y, a) = a'\}} * \left(1 - \left(\sigma(y, a) * \eta(\theta(y, a))\right)\right) * \Pi(y, y') * d\mu^o(y, a) \\ &+ \sum_{y \in Y} \int_{a \in \bar{A}} 1_{\{a'(y, a) = a'\}} * \omega(y, a) * \Pi(y, y') * d\mu^r(y, a) \end{aligned} \quad (2.13)$$

Here, $1_{\{a'(y, a) = a'\}}$ is an indicator for whether the savings policy function for an agent with state (y, a) yields saving level a' .

2.2.6 Stationary equilibrium

Stationary equilibrium consists of price index p , distributions of owners and renters $\{\mu^o, \mu^r\}$, value functions $\{V^c, V^s, v^s, V^t, V^b, V^r, V^o\}$ and associated policy functions $\{a'^r, a'^o, \theta, \omega, \sigma\}$ that satisfy the following:

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1. Owners and renters make their choices as described in section 2.2.4 given house price index p and rent ρ
2. Given $p(\theta)$, $\theta \geq 0$ and $\kappa_s \geq \alpha(\theta)(p - p(\theta))$ with complementary slackness
3. The price index equates demand and supply in the broker-intermediated housing market:

$$\int_{s^r} \omega(s^r; p) * d\mu^r(s^r) = \int_{s^o} \sigma(s^o; p) * \eta(s^o; p) * d\mu^o(s^o) \quad (2.14)$$

4. The distributions $\{\mu^j\}_{j=r,o}$ are consistent with individual sale (σ) and purchase choices (ω) and evolve as in section 2.2.5.

Condition 2 states that, given a list price function $p(\theta)$, all active submarkets entail brokers making zero profits. Implicit in the discussion in section 2.2.4 is the assumption that *all* submarkets have a list price function given by equation (2.6). In other words, inactive submarkets also have the same price function, that agents internalize when choosing tightness. If agents were to deviate to an inactive submarket, they would need to internalize that the price function in that submarket would need to satisfy the zero-profit condition for brokers while choosing the tightness level. This corresponds to the restrictions imposed on out-of-equilibrium beliefs in directed search models (see e.g. Menzio and Shi [2010], Jerez et al. [2020]). Jerez et al. [2020] employ the price-taking approach and argue that this corresponds to assuming that inactive submarkets choose the lowest price that supports the equilibrium allocation in their model.

Condition 3 requires the broker intermediated housing market to clear. The supply of houses to brokers is from matched sellers, hence the RHS of equation (2.14) includes $\eta(\theta)$. As I have assumed that buyers can trade frictionlessly with brokers, the LHS of equation (2.14) simply aggregates over all buyers.

2.3 Equilibrium sorting patterns for tightness choice

In this section, I consider the variation of market tightness with asset holdings when the non-negativity constraint on list price does not bind. In order to facilitate this analysis, it would be convenient to establish differentiability results for the value functions v^s and V^c , which is not straightforward. In appendix 2.7.1, I follow the

approach in Jerez et al. [2020], Clausen and Strub [2020] to establish differentiability results in the interior of the choice set.

Having established differentiability of the choice specific value functions, the tightness choice solves equation (2.5), which I rewrite below:

$$\max_{\theta(s_t^o) \in \Theta} \eta(\theta(s_t^o)) v^s(s_t^o) + \left(1 - \eta(\theta(s_t^o))\right) V^c(s_t^o)$$

Define the surplus from sale over repayment by $S(s_t^o) \equiv v^s(s_t^o) - V^c(s_t^o)$. The first-order condition for an interior choice of θ is:

$$\eta'(\theta(s_t^o)) S(s_t^o) + \eta(\theta(s_t^o)) v_p^s(s_t^o; p) p'(\theta) = 0 \quad (2.15)$$

I denote the variation in the matched seller's value from a change in tightness, which arises due to a change in his net worth, by $v_p^s(s_t^o; p)$. One can rewrite the above as:

$$p'(\theta(s_t^o)) = \frac{-S(s_t^o) \eta'(\theta(s_t^o))}{v_p^s(s_t^o) \eta(\theta(s_t^o))} \quad (2.16)$$

From equation (2.6), $p'(\theta) = \frac{\kappa_s}{\alpha^2(\theta)} * \alpha'(\theta)$

Substituting this into the above expression, and using the definition of the elasticity of the matching function:

$$\epsilon(\theta) \equiv \frac{\eta'(\theta) \theta}{\eta(\theta)}$$

the F.O.C for tightness becomes:

$$\frac{S(s_t^o)}{v_p^s(s_t^o)} = \frac{\left(1 - \epsilon(\theta(s_t^o))\right)}{\epsilon(\theta(s_t^o))} \left(p - p(\theta(s_t^o))\right) \quad (2.17)$$

Given $p(\theta)$, the RHS of equation (2.17) is increasing in θ . Hence, the variation of θ with asset levels depends on how the LHS varies with a . If $v^s(y, a)$ is concave in asset levels for given y , then the denominator is decreasing in a . Appendix 2.7.1.3 provides conditions under which $v^s(y, \cdot)$ is concave in a .

If the numerator is non-increasing in a for given y , as is the case numerically, then the overall sign of the relationship between θ and a will depend on which term on the

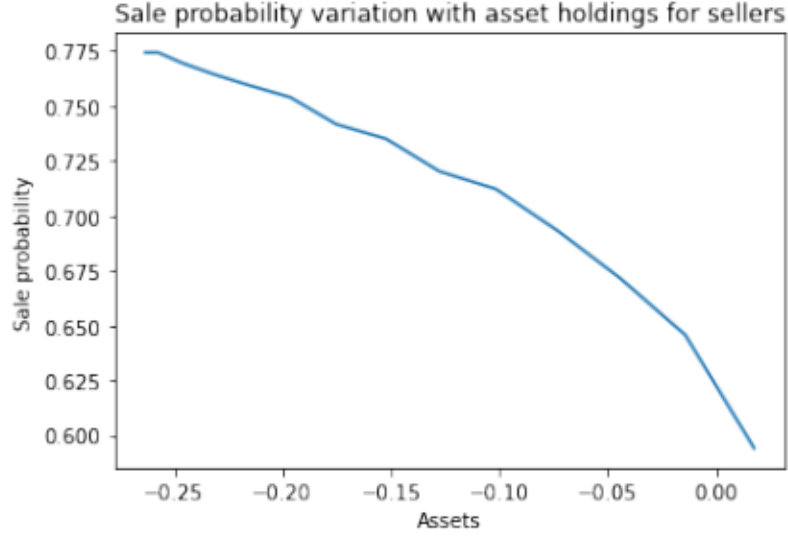


Figure 2.1: Variation of sale probability with asset holdings.

Notes: *Sellers with more assets have a favourable outside option in terms of repayment, and therefore charge higher prices while trading off lower sale probabilities.*

LHS of equation (2.17) dominates. Numerically, it turns out that the numerator of the LHS dominates, hence tightness decreases with asset level. In other words, lower debt (higher a) lowers market tightness and raises the list price. Hence, sellers with more assets have a favourable outside option in terms of repayment, and therefore charge higher prices while trading off lower sale probabilities. This can be seen in the above figure ³.

One can also show that, under some assumptions, owners choose to sell when their assets are below a certain threshold, and repay otherwise.

Proposition 1: If V^c , V^s and v^s are continuous in a and $S(y, \cdot)$ is non-increasing in a for all y , then $\exists \hat{a}(y)$ for all y such that $\sigma(y, a) = 1$ when $a < \hat{a}(y)$, and $\sigma(y, a) = 0$ otherwise.

Proof: See appendix 2.7.2.2.

³The figure is plotted using the parameter values from the first chapter of this dissertation, with the following exceptions: $\{\chi = 1.5, \beta = 0.95, \kappa = 0.1\}$. The matching function, $\eta(\theta) = (1 + \theta^{-\gamma})^{\frac{-1}{\gamma}}$, is from Jerez et al. [2020], with $\gamma = 0.65$.

2.4 Decentralized and efficient sale choice

I now consider the difference between decentralized and socially efficient sale choice, i.e. the wedge between private and the planner's sale choices.

2.4.1 Decentralized sale choice

Consider an owner with state $s^o = (y, a)$ who chooses whether to sell $\sigma(y, a) = 1$ or not. From equation (2.8), sale is chosen if:

$$V^s(y, a) > V^c(y, a)$$

Using the definition of $V^s(y, a)$ in equation (2.5), if sale probability is positive, this condition becomes equivalent to:

$$v^s(y, a) > V^c(y, a)$$

In this environment with frictional matching for sellers, owners choose to sell if their expected surplus from sale over ownership (repayment) is positive. As discussed above, this surplus is generally decreasing in asset holdings, hence if sale is chosen for a given income level, it is usually when an owner is either indebted or has low positive asset holdings.

2.4.2 Planner's sale choice and the pecuniary externality

The social planner chooses sale on behalf of an owner with state (y, a) in order to maximize a utilitarian social welfare function (SWF). The utilitarian SWF is akin to Davila et al. [2012], and can be motivated by the feature of standard incomplete markets models that agents are *ex ante* identical, prior to the realisation of idiosyncratic endowment shocks that generate heterogeneity through saving choices. Note that brokers and financial intermediaries are not included in this specification as they make zero profits (*ex ante* and realized).

The masses of owners and renters in the economy are denoted by $d\mu^o(y, a)$ and $d\mu^r(y, a)$, and are determined as part of the equilibrium. Extending the recursive

definition of [Davila et al. \[2012\]](#) to the setting here, define the Planner's problem as:

$$\begin{aligned} \Omega(\mu^o, \mu^r) = \max_{\sigma} \sum_{y \in Y} \int_{a \in \bar{A}} u(c, h) * d\mu^r(y, a) + \sum_{y \in Y} \int_{a \in \bar{A}} u(c, \chi h) * d\mu^o(y, a) \\ + \beta E \Omega(\mu'^o, \mu'^r) \end{aligned} \quad (2.18)$$

The planner decides whether or not to sell in order to maximize the SWF, taking as given the other decentralized choices of ownership and saving/borrowing. The planner internalizes the house price determination mechanism when choosing whether to sell or not. I assume that the planner's intervention is not anticipated by agents. The planner is also constrained by the market incompleteness, which arises here through the borrowing constraint faced by owners and the presence of uninsurable idiosyncratic risk by all agents in the economy. Hence, I focus on the *constrained efficient* sale choice.

It is easier to compare the planner's sale choice to the decentralized sale choice using an alternative expression for the SWF objective (in appendix 2.7.2.1) based on the definitions of choice-specific value functions. It is straightforward to show the equivalence of these two representations.

Proposition 2: The Planner chooses sale ($\sigma(y, a) = 1$) for an owner with state (y, a) if the following condition holds:

$$v^s(y, a) - V^c(y, a) + \text{Pecuniary externality} > 0 \quad (2.19)$$

Proof: See appendix 2.7.2.3.

The pecuniary externality (PE) term here is:

$$\begin{aligned} \text{PE} = \sum_{y \in Y} \int_{a \in \bar{A}} \left(\sigma(y, a) * \eta(\theta(y, a)) * u_c(c, h) * \frac{\Delta p(\theta)}{\Delta p} * d\mu^o(y, a) * \frac{\Delta p}{\Delta S} \right) \\ - \sum_{y \in Y} \int_{a \in \bar{A}} \left(\omega(y, a) * u_c(c, \chi h) * (1 + \kappa_b) * d\mu^r(y, a) * \frac{\Delta p}{\Delta S} \right) \end{aligned} \quad (2.20)$$

Pecuniary externality associated with sale choice

In this model, the pecuniary externality is *distributive* in nature. It comprises differences in marginal utilities of consumption between agents (matched sellers and buyers). These enter into the expression as matched sales increase the supply of houses, driving the price index down and thereby affecting other matched sellers and buyers in the economy. The price impact of a successful sale in equation (2.20) is represented by $\frac{\Delta p}{\Delta S} \leq 0$.

The first term in equation (2.20) is the impact of a marginal sale on matched sellers, hence the product $\sigma(y, a) * \eta(\theta(y, a))$. Further, matched sellers who are constrained by their debt position, from equation (2.2), are not affected by movements in the price index, i.e. $\frac{\Delta p(\theta)}{\Delta p} = 0$ for these sellers. Otherwise, $\frac{\Delta p(\theta)}{\Delta p} = 1$, from equation (2.6).

Distributive externalities in this environment arise when changes in net worth due to asset price changes affect buyers and matched sellers of the asset differently. With complete markets, changes in net worth would not matter as agents could perfectly hedge risk (cf. Dávila and Korinek [2018]), and the market clearing condition would imply that asset price changes would have zero aggregate impact. However, whenever agents face incomplete financial markets, they generally cannot insure perfectly against risk. This implies that, along with transaction costs incurred, distributive externalities do not wash out in the aggregate.

The market incompleteness here is due to the presence of uninsurable idiosyncratic risk and the inability to dissave beyond the credit limit. The planner would then intervene to change asset prices, by suitably changing sale intensity, so as to benefit the agents in the economy who have higher marginal utility of consumption (the agents who are more constrained). For example, if sellers are more constrained than buyers, the planner would choose to reduce sale intensity to raise prices and therefore benefit matched sellers as opposed to buyers. This can be understood from (2.20): the first term on the RHS is the pecuniary externality faced by sellers and is clearly higher, *ceteris paribus*, if the marginal utility of consumption is large (as would be the case for constrained sellers). If buyers tend to have lower marginal utilities of consumption, then the first term dominates and the overall PE term has a *negative* sign because $\frac{\Delta p}{\Delta S} \leq 0$. This implies, from (2.19) that the planner would be *less* likely to choose to sell a house.

Broker intermediation and pecuniary externalities

Generally, pecuniary externalities have been analysed in environments where the asset market is Walrasian, i.e. where asset buyers and sellers can trade frictionlessly at a price that clears the market in equilibrium. The current framework differs in that sellers do not match directly with buyers; instead, sales are intermediated by brokers. Although the broker intermediation assumption is made in order to tractably incorporate two-sided heterogeneity (by yielding block recursivity) it also provides a means to link different submarkets together through the price index that clears the overall broker-intermediated market. Matched sellers increase the stock of houses that are traded to brokers, hence with downward sloping demand by buyers, the additional supply triggers a decline in the price index. Therefore, the introduction of brokers provides a *quasi-Walrasian* market clearing price that affects the net worth of agents in active submarkets.

Distributive externalities and policy intervention

Distributive externalities generally arise in environments with market incompleteness, asset trading and agent heterogeneity. As [Dávila and Korinek \[2018\]](#) discuss, there are various, mainly theoretical, articles that rely on distributive pecuniary externalities to motivate policy intervention (e.g. [Lorenzoni \[2008\]](#), [He and Kondor \[2016\]](#), [Itskhoki and Moll \[2019\]](#)). As distributive externalities rely on differences in agents' net worth, which are affected by asset prices, the rationale for policy interventions based on correcting these externalities is to move asset prices so as to benefit more constrained agents.

2.4.3 Corrective policy

Having established the presence of an inefficiency wedge associated with sale choice, I now consider policy that could implement the efficient sale choice. The policymaker could induce agents to make the efficient sale choice through the use of many different instruments. For example, sale can be penalized through the use of a tax on cash in hand, whereas repayment could be incentivized through the use of subsidies to net worth (or reductions in debt). Here, I describe the use of such subsidies to implement

efficient sale choice. These debt reductions are only offered to owners who inefficiently choose to sell their house.

Owners who choose sale inefficiently can be made to choose repayment through a suitable subsidy policy. In particular, a potential seller with state (y, a) and continuation value $V^c(y, a)$ receives a subsidy $\tilde{a}(y, a)$ so as to implement the efficient sale choice:

$$V^c(y, \tilde{a}) = V^s(y, a)$$

As the value function is continuous and nondecreasing in a and the asset set is compact, one can use the intermediate value theorem to obtain a solution $\tilde{a}(y, a)$. As negative asset holdings correspond to debt, the subsidy policy can also be interpreted as a loan balance reduction for indebted owners who choose to sell inefficiently.

The subsidy (debt reduction) fraction is contingent on the agent state. It is greater for owners with low income and/or assets, as they require more inducement to choose efficient repayment over sale.

2.5 Extensions

The model presented above is the simplest environment in which to analyse directed search trading mechanisms that yield pecuniary externalities and inefficiencies associated with sale choice. I now discuss some possible extensions to the model and the differences in results that they generate.

2.5.1 Frictional home purchases

It is straightforward to extend the model above to frictional home purchases, so that both sides of the market operate in a directed search environment intermediated by brokers (as in [Garriga and Hedlund \[2020\]](#)).

The problem for prospective buyers becomes analogous to that for sellers, described in section 2.2.4. Let $\theta(s_t^r)$ be the *buyer's* choice of market tightness for state s_t^r , and denote by $\eta^b(\theta(s_t^r))$ and $\alpha^b(\theta(s_t^r))$ the buyer's and broker's match probabilities

respectively. Then, a buyer with state s_t^r chooses market tightness $\theta(s_t^r)$ in order to maximize his expected payoff from choosing ownership ($\omega(s_t^r) = 1$):

$$\tilde{V}^b(s_t^r) = \max_{\theta(s_t^r) \in \Theta} \eta^b(\theta(s_t^r))V^b(s_t^r) + \left(1 - \eta^b(\theta(s_t^r))\right)V^t(s_t^r) \quad (2.21)$$

subject to

$$p(\theta(s_t^r)) = p + \frac{\kappa_s}{\alpha^b(\theta(s_t^r))} \quad (2.22)$$

and

$$p(\theta(s_t^r))(1 + \kappa_b) \leq y + a$$

Thus, a prospective buyer chooses submarket tightness taking as constraints the zero profit requirement (participation constraint) for brokers, and the feasibility requirement for posted prices.

The broker-intermediated market clearing condition, which determines the equilibrium price index, becomes:

$$\int_{s^r} \omega(s^r; p) * \eta^b(s^r; p) * d\mu^r(s^r) = \int_{s^o} \sigma(s^o; p) * \eta(s^o; p) * d\mu^o(s^o) \quad (2.23)$$

Let $p^b(\theta)$ and $p^s(\theta)$ denote the bid and ask prices posted by buyers and sellers respectively. Then, the expression for the pecuniary externality associated with sale choice is readily modified to:

$$\begin{aligned} \text{PE} = & \sum_{y \in Y} \int_{a \in \bar{A}} \left(\sigma(y, a) * \eta(\theta(y, a)) * u_c(c, h) * \frac{\Delta p^s(\theta)}{\Delta p} * d\mu^o(y, a) * \frac{\Delta p}{\Delta S} \right) \\ & - \sum_{y \in Y} \int_{a \in \bar{A}} \left(\omega(y, a) * \eta^b(\theta(y, a)) * u_c(c, \chi h) * \frac{\Delta p^b(\theta)}{\Delta p} * (1 + \kappa_b) * d\mu^r(y, a) * \frac{\Delta p}{\Delta S} \right) \end{aligned} \quad (2.24)$$

2.5.2 Defaultable debt and foreclosure

I now consider how introducing defaultable debt alters the results.

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To simplify the exposition, I revert to the assumption in the baseline model above that buyers can transact frictionlessly with brokers (taking the price index as given). Default leads to lenders possessing the house. Lenders are also assumed to transact frictionlessly with brokers at the given price index⁴. However, I assume that there is a deadweight loss associated with lender ownership, so lenders receive only a fraction $\zeta < 1$ of the sale price.

This requires a modification of the problem above, as lenders now account for default risk while pricing a loan. If an owner with state (y, a) chooses borrowing level a' , the loan pricing function $Q(y, a')$ is:

$$Q(y, a') = \begin{cases} R^{-1} \mathbb{E}_{y'|y} \left[\sigma(y', a') \left(1 - \eta(\theta(y', a')) \min\left\{\frac{\zeta p'}{a'}, 1\right\} \right) + \left(1 - \sigma(y', a') \left(1 - \eta(\theta(y', a')) \right) \right) \right] & \text{if } a' < 0 \\ R^{-1} & \text{if } a' \geq 0 \end{cases}$$

This amends the value functions of buyers and owners who repay as follows:

$$V^c(y, a) = \max_{Q(y, a') a' \geq -\iota p} u(y + a - Q(y, a') a', \chi h) + \beta \mathbb{E} V^o(y', a')$$

$$V^b(y, a) = \max_{Q(y, a') a' \geq -\iota p} u\left(y + a - Q(y, a') a' - p(1 + \kappa_b), \chi h\right) + \beta \mathbb{E} V^o(y', a')$$

As written, borrowers face a collateral constraint where the maximum possible debt is ιp , and the credit constraint arises from informational and/or institutional frictions that are not modeled explicitly. Alternatively, one could have an exogenous debt limit that does not depend on the price index.

I assume that default occurs when potential sellers fail to be matched⁵. This modifies the value functions associated with potential sale to:

$$V^s(s_t^o) = \max_{\theta(s_t^o) \in \Theta} \eta(\theta(s_t^o)) v^s(s_t^o) + \left(1 - \eta(\theta(s_t^o)) \right) V^d(s_t^o)$$

subject to equation (2.6).

⁴This can be relaxed, as in Garriga and Hedlund [2020], but complicates the analysis considerably.

⁵Alternatively, one could assume that unsuccessful sales lead to a choice between default or repayment.

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Further, default is assumed to lead to permanent tenancy, and the value function associated with default is given by:

$$V^d(y, a) = \max_{a'} u(y + a - \frac{a'}{R}, h) + \beta \mathbb{E} V^d(y', a')$$

In order to derive envelope conditions in this environment, one requires a slightly different approach to that detailed in appendix 2.7.1.2. I describe the changes necessary in appendix 2.7.1.4.

Interestingly, the surplus from successful sale now generally increases in a , for a given income level. Following the argument in section 2.3, one can show that tightness choice is now increasing in asset holding. In other words, lower debt (higher a) now raises market tightness and lowers the list price. In this version of the model, sellers with more assets have a less favourable outside option in terms of default, and therefore charge lower prices while trading off higher sale probabilities.

The Social Welfare Function and sale externalities

Now, I also assume that the planner takes loan prices as given when choosing whether to sell or not. I assume that the planner's intervention is unanticipated: agents do not account for a possible intervention that would reduce default risk in the future.

The planner's problem is augmented to incorporate lender payoffs, given by:

$$\sum_{y \in Y} \int_{a \in \bar{A}} \sigma(y, a) * \left(1 - \eta(\theta(y, a))\right) * 1_{(\zeta p + a < 0)} * (\zeta p + a) * d\mu^o(y, a) \quad (2.25)$$

Hence, the planner's problem becomes:

$$\begin{aligned} \Omega(\mu^o, \mu^r) = & \max_{\{\sigma\}} \sum_{y \in Y} \int_{a \in \bar{A}} u(c, h) * d\mu^r(y, a) + \sum_{y \in Y} \int_{a \in \bar{A}} u(c, \chi h) * d\mu^o(y, a) \\ & + \sum_{y \in Y} \int_{a \in \bar{A}} \sigma(y, a) * \left(1 - \eta(\theta(y, a))\right) * 1_{(\zeta p + a < 0)} * (\zeta p + a) * d\mu^o(y, a) \\ & + \beta \mathbb{E} \Omega(\mu'^o, \mu'^r) \end{aligned} \quad (2.26)$$

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In the above, $1_{(\zeta p + a < 0)}$ is an indicator function that takes the value 1 if the lender recovery amount from sale is lower than the debt outstanding.

One can derive a version of equation (2.20) that now also accounts for *ex post* losses incurred by lenders on their loans, in a manner similar to that described in appendix 2.7.2.3.

Proposition 3: The Planner chooses sale ($\sigma(y, a) = 1$) for an owner if the following condition holds:

$$V^s(y, a) - V^c(y, a) + \text{Distributive PE} + \text{Foreclosure deadweight cost} > 0 \quad (2.27)$$

Proof: See appendix 2.7.2.3.

Then, the distributive PE term becomes:

$$\begin{aligned} \text{PE} = & \sum_{y \in Y} \int_{a \in \bar{A}} \left(\sigma(y, a) * \eta(\theta(y, a)) * u_c(c, h) * \frac{\Delta p(\theta)}{\Delta p} * d\mu^o(y, a) * \frac{\Delta p}{\Delta S} \right) \\ & + \left(\sum_{y \in Y} \int_{a \in \bar{A}} \sigma(y, a) * \left(1 - \eta(\theta(y, a)) \right) * 1_{(\zeta p + a < 0)} * \zeta * d\mu^o(y, a) * \frac{\Delta p}{\Delta S} \right) \\ & - \sum_{y \in Y} \int_{a \in \bar{A}} \left(\omega(y, a) * u_c(c, \chi h) * (1 + \kappa_b) * d\mu^r(y, a) * \frac{\Delta p}{\Delta S} \right) \end{aligned} \quad (2.28)$$

The additional term in (2.28) represents the marginal effect of a change in prices following sale on lender liquidity. As lenders are risk-neutral and assumed to be unconstrained, the marginal value of their net worth is unity. This term, when prices decline following a sale, introduces another factor that favors fewer sales.

There is an additional inefficiency associated with lender loss in the event of an unsuccessful sale, which would lead to lenders recovering $\min\{\zeta p + a, 0\}$ from selling the house to brokers (this captures the deadweight costs of a foreclosure sale). This term is:

$$\text{Foreclosure deadweight cost} = \sigma(y, a) * \left(1 - \eta(\theta(y, a)) \right) * \min\{\zeta p + a, 0\} \quad (2.29)$$

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As I discuss in the first chapter of this dissertation, renegotiation following an unsuccessful match would generally mitigate the foreclosure deadweight cost. In that sense then, this inefficiency associated with realized foreclosure losses can be interpreted as arising from frictions that impede renegotiation.

I focus on only distributive pecuniary externalities here, to relate the extension to the earlier model. If borrowers face a collateral constraint, then collateral pecuniary externalities would also arise here. These would take a form similar to the expression derived in the first chapter. One could potentially approximate the Lagrange multipliers on the collateral constraint by considering points corresponding to interior optima that are close to the bound. Collateral externalities would also favour fewer sales.

Amplification mechanism with frictional trades

The model in section 2.2 does not feature an amplification mechanism. Indeed, a decline in the price index induces potential sellers to lower their list price, which might favour more repayment by worsening the terms of trade.

With defaultable debt, there is a feedback loop between debt overhang, foreclosure and low prices. Debt overhang prevents some potential sellers from lowering their list price as they are constrained to make their required mortgage payment. This would lead to foreclosure for highly indebted owners, and a lower price index through an increase in supply. A lower price index requires list prices to be lowered further in active submarkets, which leads to more indebted sellers being foreclosed upon and so on (see Garriga and Hedlund [2020] for a further discussion).

2.5.3 Inefficient tightness choice

The presence of pecuniary externalities also affects the efficiency of the submarket tightness choice. As section 2.2 discusses, the choice of market tightness by sellers can also be interpreted as a choice of list prices.

In order to demonstrate this, I work with the baseline model of section 2.2 and set up a constrained optimization problem similar to the one described in section 2.4. The planner takes all other saving/borrowing, sale and ownership choices as given. He also internalizes the equilibrium house price determination mechanism. As before, I assume that the planner's intervention is not anticipated by agents.

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The planner chooses a tightness function so as to maximize the utilitarian SWF, subject to the broker zero-profit and consumption non-negativity constraints for potential sellers, given in equation (2.6).

Proposition 4: The planner's interior F.O.C for constrained efficient tightness choice is given by:

$$\left\{ \left(v^s(y, a) - V^c(y, a) \right) + \frac{\eta(\theta(y, a))}{\eta'(\theta(y, a))} v_p^s(y, a) p'(\theta) \right\} + PE_\theta = 0 \quad (2.30)$$

Proof: See appendix 2.7.2.5.

The pecuniary externality (PE_θ) associated with tightness choice is:

$$\begin{aligned} PE_\theta = & \left\{ - \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') * (u_c(c, \chi h)) * d\mu^r(y', a') \right. \\ & \left. + \sum_{y' \in Y} \int_{a' \in \bar{A}} \left(\sigma(y', a') * \eta(\theta(y', a')) * u_c(c, h) \right) * \frac{dp(\theta(y', a'))}{dp} * d\mu^o(y', a') \right\} \\ & * \frac{dp}{dS} \quad (2.31) \end{aligned}$$

This is virtually identical to the expression in equation (2.20), which is unsurprising as the channel through which the inefficiency arises is the price index. However, a marginal change in $\theta(y, a)$ changes sale probability by $\eta'(\theta(y, a))d\theta$, and changes supply by $\sigma(y, a) * \eta'(\theta(y, a)) * d\mu^o(y, a) * d\theta$. This is captured in the expression above by the term $\frac{dp}{dS}$.

If $PE_\theta < 0$, then the Planner would reduce tightness and sale probability (as $\eta' > 0$) relative to the decentralized tightness choice. This is because higher market tightness increases the probability of a successful sale, and more sales would lower the equilibrium price index p and thereby adversely affect other agents in the economy. The condition that $PE_\theta < 0$ would tend to hold when sellers are more constrained than buyers. The reasoning is analogous to the case in section 2.4.2: if sellers are more constrained on average than buyers, they tend to have greater marginal utilities of consumption than buyers and hence the overall term in braces in equation (2.31) is positive. Since $\frac{dp}{dS} < 0$, so is PE_θ .

One could alternatively interpret the result on market tightness in terms of list prices. As list prices are inversely related to market tightness, one could also state that, if sellers tend to be more constrained as a group than buyers, then private list prices are lower than the efficient list price benchmark.

Hence, in this model with directed search and choice of submarket tightness, pecuniary externalities due to incomplete markets and frictions lead to inefficient private submarket tightness choices. On this basis, one may conjecture that a similar inefficiency would arise in other directed search models that feature market incompleteness (such as a collateral constraint that may bind, or imperfect risk sharing) and, importantly, link various submarkets together through an intermediation mechanism such as the brokers considered here. Although brokers allow a tractable analysis of two-sided heterogeneous matching problems, a directed search model without brokers that features a collateral constraint would still require a specification of the (aggregate) price index that enters into that constraint. If the aggregate price is affected by individual tightness (or alternatively, list price) choices, then one would expect a pecuniary externality that would similarly lead to inefficient tightness choices.

2.6 Conclusion

This paper uses a simple incomplete markets housing model with sale/repayment choice and intermediated frictional trades in order to study the constrained inefficiency of house sale decisions. The specific assumptions of broker intermediation and quasi-Walrasian market clearing in the broker-intermediated market link various submarkets together, thereby introducing a pecuniary externality channel. This particular model feature, which also facilitates analysis of a two-sided heterogeneous agent matching problem, allows one to extend inefficiency results obtained in frictionless housing models to an environment with matching frictions in the housing market.

In addition, I show that the choice of market tightness is also constrained inefficient in this setup, owing to the pecuniary externalities associated with tightness choice and sale probability. This pecuniary externality based inefficiency result has not, to my knowledge, been discussed previously in the directed search literature.

Extending this finding to specific environments with financial frictions and frictional trading, such as OTC markets, would be an interesting next step. Developing the

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model to allow for default choice and collateral constraints would bring the model closer to the quantitative literature, e.g. [Garriga and Hedlund \[2020\]](#). This would also facilitate an analysis of the magnitude of the pecuniary externality-based inefficiencies and associated corrective policies.

2.7 Appendix to chapter 2

Appendix 2.7.1 establishes differentiability results for the choice-specific value functions, and Appendix 2.7.2 contains proofs of the propositions and results in the paper.

2.7.1 Differentiability and concavity of value functions

In this appendix, I establish differentiability of the agents' value functions. In doing so, one cannot directly apply the results of Clausen and Strub [2012] to nonconcave problems, for the reasons described in Menzio et al. [2013], Jerez et al. [2020]. In particular, in order to show the differentiability of V^s , which from equation (2.5) is a convex combination of v^s and V^c that also depends on policy functions, one needs to show the differentiability of these two value functions. Hence, it does not fall within the structure of the problems analysed by Clausen and Strub [2012]. I use a related approach based on subdifferentials, which I now describe.

2.7.1.1 Fréchet sub- and superdifferentials

This appendix establishes differentiability of choice specific value functions along interior optima. In order to do so, I use the concepts of Fréchet sub- and superdifferentials, which I refer to alternatively as F sub- and superdifferentials. Below, I define and state some properties of Fréchet sub- and superdifferentials, that I shall employ in the proofs later. The exposition below is based heavily on Jerez et al. [2020] and Clausen and Strub [2012, 2020]. An alternative definition of Fréchet differentials based on limits superior and inferior can be found in Clausen and Strub [2012]. The two definitions are related in appendix F of Clausen and Strub [2016].

For a continuous function $f : \Omega \subset \mathbb{R}^n \rightarrow \mathbb{R}$, where Ω is an open set, the vector $p \in \mathbb{R}^n$ belongs to the F-superdifferential of f at $x_0 \in \Omega$, $D^+f(x_0)$, if and only if there exists a continuous function $\phi : \Omega \rightarrow \mathbb{R}$ which is differentiable at x_0 with $D\phi(x_0) = p$, $f(x_0) = \phi(x_0)$ and $f - \phi$ has a local maximum at x_0 .

Similarly, $p \in \mathbb{R}^n$ belongs to the F-subdifferential of f at $x_0 \in \Omega$, $D^-f(x_0)$, if and only if there exists a continuous function $\phi : \Omega \rightarrow \mathbb{R}$ which is differentiable at x_0 with $D\phi(x_0) = p$, $f(x_0) = \phi(x_0)$ and $f - \phi$ has a local minimum at x_0 . $D^+f(x_0)$

and $D^-f(x_0)$ are closed convex subsets of \mathbb{R} . If f is differentiable at x_0 , then both $D^+f(x_0)$ and $D^-f(x_0)$ are nonempty and $D^+f(x_0) = D^-f(x_0) = Df(x_0)$.

Conversely, if for a function f , both $D^+f(x_0)$ and $D^-f(x_0)$ are nonempty, then f is differentiable at x_0 and $D^+f(x_0) = D^-f(x_0) = Df(x_0)$, where Df denotes the derivative of f . Finally, whenever x_0 is a local maximum of f in Ω , $0 \in D^+f(x_0)$.

The following properties of F-sub and superdifferentials will be useful below. Let f and g be functions from $\mathbb{R} \rightarrow \mathbb{R}$.

1. If f and g are Fréchet sub (super) differentiable, then so is $f + g$ (cf. [Clausen and Strub \[2012\]](#), Lemma 2(i)).
2. If $h(x) = \max\{f(x), g(x)\}$ is differentiable at \bar{x} and $f(\bar{x}) = h(\bar{x})$, then f is differentiable at \bar{x} (cf. [Clausen and Strub \[2020\]](#), Lemma 2(iii)).
3. If f and g are subdifferentiable, and $f + g$ is superdifferentiable, then f, g and $f + g$ are differentiable (cf. [Clausen and Strub \[2012\]](#), Lemma 2(iii)).

One can generalize (3) above to the case of finite sums in order to obtain envelope results for stochastic dynamic programming problems, where one takes a convex combination of value functions using the Markov probabilities, as discussed after the statement of Theorem 3 of [Clausen and Strub \[2012\]](#). In addition, I shall sometimes use the results of Lemma 2 of [Clausen and Strub \[2020\]](#) on 'Reverse Calculus'.

Finally, I state the following theorem that shall be used to establish the differentiability of value functions. Let $f(x) = \max_{y \in \Gamma(x)} F(x, y)$, where $F : X \times Y \rightarrow \mathbb{R}$ is continuous, $X, Y \subset \mathbb{R}^n$, and where Γ is a nonempty, compact valued and continuous correspondence from X to Y .

Theorem 1: Let x_0 be an interior point of X and $y_0 \in \Gamma(x_0)$ satisfying: (i) $f(x_0) = F(x_0, y_0)$, (ii) there is a ball $B(x_0, \epsilon)$ in X with center x_0 and radius $\epsilon > 0$ such that $\forall x \in B(x_0, \epsilon), y_0 \in \Gamma(x)$. Then $D_x^-F(x_0, y_0) \subseteq D^-f(x_0)$ and $D^+f(x_0) \subseteq D_x^+F(x_0, y_0)$, where $D_x^\pm F(x_0, y_0)$ denotes the F-super/subdifferential of the function $F(x, y_0)$.

Proof: See [Jerez et al. \[2020\]](#).

2.7.1.2 Differentiability of the value functions

As in Jerez et al. [2020], I establish differentiability of value functions by showing that the F sub- and superdifferentials are nonempty along the optimal policies. I assume that there exist interior selections of saving/borrowing policies $\forall y$, denoted by $g_t^a, g_n^a, g_c^a, g_s^a$ denote the relevant saving/borrowing choices for tenants, new buyers, owners who continue and owners who sell respectively. I also assume that an interior selection of the tightness choice exists, denoted by g_s^θ . The set of all feasible asset positions is \bar{A} , with minimum asset holding value denoted by \underline{a} , which is negative.

Lemmas 1 and 2 establish F-subdifferentiability of the value functions, while Lemma 3 and the subsequent discussion establishes F-superdifferentiability of the value functions.

Lemma 1: Let $a_0 > \underline{a}$. Then, $\forall y$ (i) $u_c(y + a_0 - \rho - \frac{g_t^a(y, a_0)}{R}, h) \in D_a^- V^t(y, a_0)$; (ii) $u_c(y + a_0 - \frac{g_n^a(y, a_0)}{R}, \chi h) \in D_a^- V^c(y, a_0)$; (iii) $u_c(y + a_0 - \frac{g_n^a(y, a_0)}{R}, \chi h) \in D_a^- V^b(y, a_0)$; (iv) $u_c(y + a_0 - \rho - \frac{g_s^a(y, a_0)}{R}, h) \in D_a^- v^s(y, a_0)$

Proof. I only prove (i), as the proofs of the other cases are almost identical, mutatis mutandis. As a_0 is interior, condition (i) of Theorem 1 is satisfied. Given that $g_t^a(y, a_0)$ is interior and the feasible correspondence is closed, there is an open interval centered at a_0 such that $g_t^a(y, a) \in (0, R(y + a))$ for all a in this open interval. Hence, condition (ii) of Theorem 1 is satisfied.

Construct the function:

$$F(y, a, g_t^a(y, a_0)) = u(y + a - \frac{g_t^a(y, a_0)}{R}, h) + \beta EV^r(y', g_t^a(y, a_0))$$

Clearly, this function is differentiable w.r.t a as the second term does not depend on a . The derivative w.r.t a at a_0 is $u_c(y + a_0 - \frac{g_t^a(y, a_0)}{R}, h)$. Hence, by Theorem 1, $u_c(y + a_0 - \frac{g_t^a(y, a_0)}{R}, h) \in D_a^- V^t(y, a_0)$.

□

In the case of a seller choosing market tightness, I denote the optimal choice of θ given (y, a) by $g_s^\theta(y, a)$. As the domain of choice for θ ,

$$D(a) = \{\theta \in \mathbb{R}_+ : p(\theta) \geq 0\}$$

is not compact, I follow Jerez et al. [2020] and transform the seller's choice of tightness into a choice of sale probability, η . One can then express the list price in terms of η :

$$\hat{p}(\eta) = p - \frac{\kappa_s}{\hat{\alpha}(\eta)}; \eta \in (0, 1)$$

and the domain is modified to:

$$\hat{D}(a) = \{\eta \in [0, 1] : \hat{p}(\eta) \geq 0\}$$

The sections of \hat{D} are nonempty and compact for $\hat{p}(\eta) + a + y > 0$. One can then rewrite the seller's problem as:

$$\hat{V}^s(y, a) = \max_{\eta} \eta v^s(y, a) + (1 - \eta) V^c(y, a)$$

and the optimal choice of η for owner with state (y, a) is $g_s^\eta(y, a)$.

Lemma 2: Let $a_0 > \underline{a}$ and suppose $\exists \tilde{a}(y)$ given y such that $\sigma(y, a) = 1$ if $a < \tilde{a}(y)$, and $\sigma(y, a) = 0$ otherwise. Then, (i) if $a_0 < \tilde{a}(y)$, then $\left(1 - \eta(\theta(y, a_0))\right) u_c(y + a_0 - \frac{g_s^a(y, a_0)}{R}, \chi h) + \eta(\theta(y, a_0)) u_c(y + a_0 - \rho - \frac{g_s^a(y, a_0)}{R}, h) \in D_a^- V^o(y, a_0)$; (ii) if $a_0 \geq \tilde{a}(y)$, then $u_c(y + a_0 - \frac{g_s^a(y, a_0)}{R}, \chi h) \in D_a^- V^o(y, a_0)$.

Proof. For case (ii), $V^o(y, a_0) = V^c(y, a_0)$ and the result follows from Lemma 1, part (ii). For case (i), as g_s^θ is interior, the optimal g_s^η is also interior. Construct the function:

$$F(y, a, g_s^\eta(a_0)) = g_s^\eta(a_0) v^s(y, a) + (1 - g_s^\eta(a_0)) V^c(y, a)$$

which is well-defined in an open interval around a_0 . Now, consider $p_s \in D_a^- v^s(y, a_0)$ and $p_c \in D_a^- V^c(y, a_0)$, which exist by Lemma 1 and are equal to $u_c(y + a_0 - \rho - \frac{g_s^a(y, a_0)}{R}, h)$ and $u_c(y + a_0 - \frac{g_s^a(y, a_0)}{R}, \chi h)$ respectively. Further, a convex combination of these subdifferentials belongs in $D_a^- F(y, a_0, g_s^\eta(a_0))$, i.e.

$$\eta(g_s^\theta(y, a_0)) p_s + (1 - \eta(g_s^\theta(y, a_0))) p_c \in D_a^- F(y, a_0, g_s^\theta(a_0))$$

From Theorem 1, $D_a^- F(y, a_0, g_s^\eta(a_0)) \subseteq D_a^- V^o(y, a_0) = D_a^- V^s(y, a_0)$. □

One can similarly show that the subdifferential of V^b is nonempty.

I now establish nonemptiness of the superdifferentials of value functions.

Lemma 3: Let $a_0 > \underline{a}$. Then, $\forall y, u_c(y + a_0 - \frac{g_c^a(y, a_0)}{R}, \chi h) \in \beta RD_a^+ EV^o(y', g_c^a(y, a_0))$

Proof. Consider the following function of a' :

$$F(y, a_0, a') = u(y + a_0 - \frac{a'}{R}, \chi h) + \beta E_{y'} V^o(y', a')$$

As $g_c^a(y, a_0)$ is an interior optimum for the $V^c(y, a_0)$, $0 \in D_a^+ F(y, a_0, g_c^a(y, a_0))$. As u is C^1 , $D_a^+ F = \frac{-u_c}{R} + \beta D_a^+ EV^o$. Thus, $u_c(y + a_0 - \frac{g_c^a(y, a_0)}{R}, \chi h) \in \beta RD_a^+ EV^o(y', g_c^a(y, a_0))$. \square

Proposition A1: Let $a_0 > \underline{a}$. Then, $\forall y', V^o(y', a')$ is differentiable w.r.t a at $a' = g_c^a(y, a_0)$. Further, $V^s(y', a')$ and $V^c(y', a')$ are also differentiable w.r.t a at $a' = g_c^a(y, a_0)$.

Proof. As $V^o(y, a_0) = \max\{V^c(y, a_0), V^s(y, a_0)\}$, it is subdifferentiable if $a_0 > \underline{a}$, from Lemma 1. Since $g_c^a(y, a_0)$ is interior, it exceeds \underline{a} .

Using Lemma 3 and applying property (3) of F sub-differentials in appendix 2.7.1.1 extended to convex combinations using Markov probabilities of $V^o(\cdot, a')$, $V^o(\cdot, a')$ is differentiable in a' at $g_c^a(y, a_0)$.

Applying property (2) in appendix 2.7.1.1, $V^c(y', g_c^a(y, a_0))$ and $V^s(y', g_c^a(y, a_0))$ are also differentiable w.r.t a at $g_c^a(y, a_0)$, depending on whether $\sigma(y', g_c^a(y, a_0)) = 1$ or not. \square

Similarly, one can establish the differentiability of $V^r(y', \cdot)$ w.r.t a at $g_t^a(y, a_0)$, and hence, by applying property (2) in appendix 2.7.1.1 again, $V^t(y', g_t^a(y, a_0))$ and $V^b(y', g_t^a(y, a_0))$ are also differentiable w.r.t a at $g_t^a(y, a_0)$, depending on whether $\omega(y', g_t^a(y, a_0)) = 1$ or not.

Further, if $a > \underline{a}$, Lemma 1 derived the F-subdifferentials of V^t and V^b , and as $V^r(y, a) = \max\{V^t(y, a), V^b(y, a)\}$, it is also subdifferentiable. By the same argument

as in Lemma 3, one can show that $u_c(y + a_0 - \rho - \frac{g_s^a(y, a_0), h}{R}) \in \beta RD_d^+ EV^r(y', g_s^a(y, a_0))$. Hence, an application of property (3) of F-subdifferentials to convex combinations of $V^r(\cdot, a')$ in appendix 2.7.1.1 implies that $V^r(y', \cdot)$ is differentiable w.r.t a at $g_s^a(y, a_0)$.

2.7.1.3 Concavity of the value functions

Concavity of the value function v^s is proved in intervals of the image of $g_s^a(y, a)$ corresponding to sale such that seller consumption is nondecreasing in the range of assets that lead to sale. Denote optimal consumption of a seller by the policy function $g_s^c(y, a)$.

Proposition A2: $\forall y, E_{y'|y} V^r$ is concave in a in the intervals I of the image of g_s^a iff g_s^c is nondecreasing in $(g_s^a)^{-1}(I)$.

Proof. From the argument above, V^r is differentiable w.r.t a in I for all y . Fix a value for y . If $a' \in I$, then $\exists a > \underline{a}$ such that $g_s^a(y, a) = a'$ and $E_{y'|y} V_a^r(y', g_s^a(y, a)) = \frac{u_c(g_s^c(y, a), h)}{\beta R}$. Let $(a'_1, a'_2) \in I$ s.t. $a'_1 > a'_2$, and suppose $\exists a_i$ such that $a'_i = g_s^a(a_i)$ for $i = 1, 2$.

By the Mean value Theorem,

$$E_{y'|y} V^r(y', a'_1) - E_{y'|y} V^r(y', a'_2) = E_{y'|y} V_a^r(y', b')(a'_1 - a'_2) = \frac{u_c(g_s^c(y, b), h)}{\beta R}(a'_1 - a'_2)$$

where $a'_2 < b' < a'_1$ and $b' = g_s^a(y, b)$.

Given y , $W(y, a, a') = u(y, a, h; a') + \beta EV^r(y', a')$ satisfies increasing first differences in (a, a') as $u(y, a, h; a')$ satisfies increasing first differences in (a, a') , hence by Theorem 10.6 of Sundaram [1996], $g_s^a(y, \cdot)$ is nondecreasing in a . Thus, $a_2 < b < a_1$. As $g_s^c(y, \cdot)$ is nondecreasing in a by assumption, we also have $g_s^c(y, a_2) < g_s^c(y, b) < g_s^c(y, a_1)$. As $u(\cdot, h)$ is concave, $u_c(g_s^c(y, b), h) \leq u_c(g_s^c(y, a_2), h) = R\beta E_{y'|y} V_a^r(y', a'_2)$.

Hence,

$$E_{y'|y} V^r(y', a'_1) - E_{y'|y} V^r(y', a'_2) \leq E_{y'|y} V_a^r(y', a'_2)(a'_1 - a'_2)$$

and so $E_{y'|y}V^r(y', \cdot)$ is concave in I . The proof of the converse follows the reverse direction. \square

Proposition A3: Let I be an interval of \bar{A} such that $g_s^a(y, I)$ is an interval, $\forall y$. Then, v_s is concave in I iff $g_s^c(y, \cdot)$ is nondecreasing in I .

Proof. Let $a_1, a_2 \in I$, and let $\lambda_1, \lambda_2 \in [0, 1]$ such that $\lambda_1 + \lambda_2 = 1$. Fix a level of y . As $g_s^a(y, I)$ is convex, $\lambda_1 a_1 + \lambda_2 a_2 \in I$, and $\lambda_1 g_s^a(y, a_1) + \lambda_2 g_s^a(y, a_2) \in g_s^a(y, I)$.

Further, $(\lambda_1 a_1 + \lambda_2 a_2, \lambda_1 g_s^a(y, a_1) + \lambda_2 g_s^a(y, a_2))$ belongs to the graph of the feasible correspondence for a seller, as it is convex. Finally, $V^r(y, \cdot)$ is concave in $g_s^a(y, I)$ from Proposition A2.

Then,

$$\begin{aligned} v^s(y, \lambda_1 a_1 + \lambda_2 a_2) &\leq u(\lambda_1 a_1 + \lambda_2 a_2, \lambda_1 g_s^a(y, a_1) + \lambda_2 g_s^a(y, a_2), h) + \beta EV^r(y', \lambda_1 g_s^a(y, a_1) + \lambda_2 g_s^a(y, a_2)) \\ &\leq \lambda_1 u(a_1, g_s^a(y, a_1), h) + \lambda_2 u(a_2, g_s^a(y, a_2), h) + \beta EV^r(y', \lambda_1 g_s^a(y, a_1)) + \beta EV^r(y', \lambda_2 g_s^a(y, a_2)) \\ &= \lambda_1 v^s(y, a_1) + \lambda_2 v^s(y, a_2) \end{aligned}$$

The second inequality follows from the concavity of u and the concavity of $EV^r(y', \cdot)$ in the image of $g_s^a(y, a)$. Hence, $v^s(y, \cdot)$ is concave in I . \square

2.7.1.4 Differentiability when debt is defaultable

As discussed in section 2.5.2, allowing for defaultable debt introduces a loan pricing function $Q(y, a')$ that is not necessarily differentiable. This complicates our differentiability results above, as the proof of nonemptiness of F-superdifferentials for owner value function V^o in Lemma 3 requires Q to be differentiable at the interior optimum. Hence, one requires a slightly different approach.

In the following, I show that $Q(y, \cdot)$ is differentiable using the techniques of Clausen and Strub [2020], particularly those utilized in Theorem 3 of that paper.

Subdifferentials and differentiable lower support functions

One can relate the differentiable lower support function L discussed in Clausen and Strub [2020] to the function ϕ described in the definition of F-subdifferentials in appendix 2.7.1.1.

L is a differentiable lower support function for a function f at point \bar{c} if (i) $f(c) \geq L(c) \forall c$, (ii) $f(\bar{c}) = L(\bar{c})$, and (iii) L is differentiable at \bar{c} .

If L is continuous, then clearly $f(c) - L(c) \geq f(\bar{c}) - L(\bar{c}) = 0$, so $f - L$ has a local minimum at \bar{c} . Then, L is a lower support function for f at \bar{c} , and $DL(\bar{c}) \in D^- f(\bar{c})$

Differentiability of Q w.r.t a' at an interior optimum

Lemma 1 of Clausen and Strub [2020] shows that if a function f has lower and upper support functions that are differentiable at point \bar{c} , then f is differentiable at \bar{c} .

In order to show that $Q(\cdot, a')$ is differentiable w.r.t a' , I sketch out how to construct suitable lower and upper support functions below. The exposition is based on section 3.2 of Clausen and Strub [2020].

Let a' be the asset holding choice for an owner with state (y, a) who repays. Define function $\Phi(a'; y, a)$ as follows:

$$\Phi(a'; y, a) = u(y + a - Q(y, a')a', \chi h) + \beta EV^o(y', a')$$

Lower support function

First, I construct a lower support function for $\Phi(a'; y, a)$. For simplicity, I assume that $y \in [\underline{y}, \bar{y}]$ with Markov transition function $G(y, \cdot)$. Assume that there is a threshold function defined for each asset level, $y(a)$ such that sale is chosen when $y < y(a)$, and repayment is chosen when $y \geq y(a)$. One can then write:

$$V^o(y', a') = \int_{\underline{y}}^{y(a')} V^s(y', a') dG(y, y') + \int_{y(a')}^{\bar{y}} V^c(y', a') dG(y, y')$$

and

$$Q(y, a') = R^{-1} * \left\{ 1 + \int_{\underline{y}}^{y(a')} \eta(\theta(y', a')) \left(\min\left\{ \frac{\chi p'}{a'}, 1 \right\} - 1 \right) dG(y, y') \right\}$$

In order to obtain a lower support function for $\Phi(., y, a)$ at \bar{a}' , one requires a differentiable upper support function for $y(.,)$, and lower support functions for $\theta(y', .)$ and $V^o(y', .)$ at \bar{a}' .

For the upper support function, consider an owner choosing to save or borrow with state (y', \bar{a}') who incorrectly perceives the state to be $(y(\bar{a}'), \bar{a}')$, i.e. he expects his income to be at the threshold level, and chooses asset holding \bar{a}'' accordingly. His value function is:

$$L(y', a'; \bar{a}') = u(y' + a' - Q(y', \bar{a}''), \chi h) + \beta EV^o(y'', \bar{a}'')$$

Similarly, one can construct a value function for an owner who decides to sell who also perceives the state to be $(y(\bar{a}'), \bar{a}')$. Let us denote this by $\tilde{S}(y(\bar{a}'), \bar{a}')$.

His cutoff between sale and repayment is then defined implicitly by:

$$L(\bar{y}(a'; \bar{a}'), a'; \bar{a}') = \tilde{S}(\bar{y}(a'; \bar{a}'), a'; \bar{a}')$$

This provides a cutoff $\bar{y}(.,.)$ for $y(.,.)$ that involves selling too often. Since L and \tilde{S} are differentiable w.r.t a' , so is $\bar{y}(a'; \bar{a}')$.

In order to obtain lower support functions for V^o and Q , consider an owner who perceives the state to be (y', \bar{a}') instead of (y', a') . He chooses asset holding \bar{a}'' and tightness θ accordingly.

Define the following functions:

$$M^S(y', a'; \bar{a}') = \eta(\theta(y', \bar{a}')) M^s(y', \bar{a}') + (1 - \eta(\theta(y', \bar{a}'))) M^c(y', \bar{a}')$$

where

$$M^s(y', a'; \bar{a}') = u\left(y + a' - Q(y, \bar{a}'') \bar{a}'' + p(\theta(y', \bar{a}')), h\right) + \beta EV^r(y'', \bar{a}'')$$

$$M^c(y', a'; \bar{a}') = u\left(y + a' - Q(y, \bar{a}'')\bar{a}'', \chi h\right) + \beta EV^o(y'', \bar{a}'')$$

Also, define:

$$M^o(y', a'; \bar{a}') = \int_{\underline{y}}^{\bar{y}(a'; \bar{a}')} M^s(y', a'; \bar{a}') dG(y, y') + \int_{\bar{y}(a'; \bar{a}')}^{\bar{y}} M^c(y', a'; \bar{a}') dG(y, y')$$

and

$$\underline{Q}(y, a'; \bar{a}') = R^{-1} * \left\{ 1 + \int_{\underline{y}}^{\bar{y}(a'; \bar{a}')} \eta(\theta(y', \bar{a}')) \left(\min\left\{\frac{\chi p'}{a'}, 1\right\} - 1 \right) dG(y, y') \right\}$$

Then, $\underline{Q}(y, .; \bar{a}')$ is a differentiable lower support function at \bar{a}' , and so is $M^o(y', .; \bar{a}')$ for all y' and \bar{a}' .

Hence, the function of a' :

$$\underline{\Phi}(a'; y, a) = u(y + a - \underline{Q}(y, a'; \bar{a}')a', \chi h) + \beta EM^o(y', a'; \bar{a}')$$

is a lower support function for $\Phi(.; y, a)$ at \bar{a}' .

Note the similarity of the constructions M^s , M^c and Q to the function F in the proof of Lemma 1 in appendix 2.7.1.2.

Upper support function

Define function U for an arbitrary y as follows:

$$U(a'; y, a) = u(y + a - Q(y, a')a', \chi h) + \beta EV^o(y', a')$$

The upper support function at \bar{a}' is then the constant function $U(\bar{a}'; y, a)$.

If $\bar{a}'(.; .)$ is the optimal policy and $\bar{a}'(y, a) = \bar{a}'$, then $\Phi(.; y, a)$ has differentiable upper and lower support functions at \bar{a}' , so by Lemma 1 of Clausen and Strub [2020], it is differentiable at \bar{a}' .

CHAPTER 2. CONSTRAINED INEFFICIENCY IN DIRECTED SEARCH MODEL

Now, repeatedly apply Lemma 2 of Clausen and Strub [2020] as follows. Using the summation property of their Lemma 2 (i), $u(y + a - Q(y, a')a', \chi h)$ is differentiable at \bar{a}' . Next, apply part (iv) of their Lemma 2 to $a' \rightarrow u(y + a - Q(y, a')a', \chi h)$, which establishes that $a' \rightarrow Q(a')a'$ is differentiable at \bar{a}' . Finally, apply part (ii) of their Lemma 2 to establish that $Q(y, \cdot)$ is differentiable at \bar{a}' .

Having thus shown the differentiability of $Q(y, \cdot)$ at the interior optimum for a given y , we can then modify the proof of Lemma 3 in order to establish differentiability of value functions at interior optima.

2.7.2 Alternative SWF and proofs of propositions

2.7.2.1 Alternative SWF

The transition operator for distribution μ^j is given by $\mu'^j = T\left(\mu^j, \mu^{-j}, Q^j(s^j, s'^j)\right)$, where $Q(., ., .)$ is the transition matrix from state s^j to s'^j for an individual agent and $T(., ., .)$ is the updating operator. Below, I do not use the superscript j to differentiate between the state vector and state space S^j for owners and renters, to avoid further cumbersome notation. I abuse notation in order to be concise by aggregating over the sum space using the summation symbol. The consumption, saving and loan choices are policy functions, as the use of the respective value functions indicates.

Aggregate welfare is:

$$\begin{aligned} \mathcal{W} = & \sum_{y \in Y} \int_{\bar{A}} \left(u(c, h) + \sum_{s' \in S'} \beta \left[\omega(y, a) * V^o(y', a') * Q^o(s, s') \right. \right. \\ & \left. \left. + (1 - \omega(y, a)) * V^r(y', a') * Q^r(s, s') \right] \right) * d\mu^r(y, a) \\ & + \left(\sum_{y \in Y} \int_{\bar{A}} u(c, \chi h) + \sum_{s' \in S'} \beta \left[\sigma(y, a) * \eta(\theta(y, a)) * V^r(y', a') * Q^r(s, s') + \right. \right. \\ & \left. \left. (1 - \sigma(y, a) * (1 - \eta(\theta(y, a)))) * V^o(y', a') * Q^o(s, s') \right] \right) * d\mu^o(y, a) \end{aligned} \quad (2.32)$$

2.7.2.2 Proposition 1: proof

Proposition 1: If V^c , V^s and v^s are continuous in a , $S(y, a)$ is non-increasing in a , then $\exists \hat{a}(y)$ such that $\sigma(y, a) = 1$ when $a < \hat{a}(y)$, and $\sigma(y, a) = 0$ otherwise.

Proof. $V^s(y, a) = \max\{v^s(y, a), V^c(y, a)\}$, and at \bar{a} , $V^s(y, \bar{a}) = v^s(y, \bar{a})$. As V^s , v^s and V^c are differentiable in a , one can apply the envelope theorem to obtain:

$$V_a^s(y, a) - V_a^c(y, a) = \eta(\theta(y, a)) \left(v_a^s(y, a) - V_a^c(y, a) \right)$$

The RHS of the above equation is negative, as η is positive and $S(y, a)$ is decreasing in a given y under the assumption made. Thus, $V^s(y, a) - V^c(y, a)$ is decreasing in a . If $V^s(y, \underline{a}) - V^c(y, \underline{a}) < 0$, then $\hat{a}(y) = \underline{a}$, and if $V^s(y, a^{\max}) - V^c(y, a^{\max}) > 0$, then $\hat{a}(y) = a^{\max}$. Otherwise, as the value functions are continuous in a , by the intermediate value theorem, there $\exists \hat{a}(y)$ such that $V^o(y, a) = V^s(y, a)$ for all $a < \hat{a}(y)$, and $V^o(y, a) = V^c(y, a)$ otherwise.

□

2.7.2.3 Sale choice

Proposition 2: The Planner chooses sale ($\sigma(y, a) = 1$) for an owner if the following condition holds:

$$v^s(y, a) - V^c(y, a) + \text{PE} > 0 \quad (2.33)$$

where:

$$\begin{aligned} \text{PE} = & \sum_{y \in Y} \int_{a \in \bar{A}} \left(\sigma(y, a) * \eta(\theta(y, a)) * u_c(c, h) * \frac{\Delta p(\theta)}{\Delta p} * d\mu^o(y, a) * \frac{\Delta p}{\Delta S} \right) \\ & - \sum_{y \in Y} \int_{a \in \bar{A}} \left(\omega(y, a) * u_c(c, \chi h) * (1 + \kappa_b) * d\mu^r(y, a) * \frac{\Delta p}{\Delta S} \right) \end{aligned}$$

Proof. The approach here is to use a perturbation argument in the spirit of [Davila et al. \[2012\]](#). Throughout the proof, I use the expression for the SWF in equation (2.32).

Let the Planner's sale choice be $\sigma(y, a)$. Consider a perturbation where the positive mass $d\mu^o(y, a)$ of owners with state (y, a) for whom the unique optimal choice is to sell are now switched to continuation status, i.e. they switch from $\sigma(y, a) = 1$ to $\sigma(y, a) = 0$. This reduces housing supply by an amount $\Delta S = \eta(\theta(y, a)) * d\mu^o(y, a)$, and I denote the resulting price change by $\Delta p \geq 0$. Finally, let the change in felicity of an agent due to a price change be denoted by $\Delta_p u(c, h; p)$.

The resulting change in social welfare from the perturbation, denoted by ΔW , is:

$$\Delta W = \left(V^c(y, a) - V^s(y, a) \right) * d\mu^o(y, a)$$

$$\begin{aligned}
& + \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') \left(\Delta_p u(c, \chi h; p) \right) * d\mu^r(y', a') \\
& + \sum_{y' \in Y} \int_{a' \in \bar{A}} \left(\sigma(y', a') * \eta(\theta(y', a')) * \Delta_p u(c, h; p) \right) * d\mu^o(y', a')
\end{aligned}$$

Using the first order approximation $u'(c)\Delta c \approx \Delta u$, consider for example the change in utility of sellers due to the price change. Seller consumption is $c = y + a + p(\theta) - \frac{a'}{R} - \rho$, so $\Delta c = \frac{\Delta p(\theta)}{\Delta p} * \Delta p$. Hence, $\Delta_p u(c, h; p) = u_c(c, h; p) * \frac{\Delta p(\theta)}{\Delta p} * \Delta p = -u_c(c, h; p) * \frac{\Delta p(\theta)}{\Delta p} * \frac{\Delta p}{\Delta S} * \eta(\theta(y, a)) * d\mu^o(y, a)$. Similarly, for buyers, the change in utility is $\Delta_p u(c, \chi h; p) = u_c(c, \chi h; p) * (1 + \kappa_b) * \frac{\Delta p}{\Delta S} * \eta(\theta(y, a)) * d\mu^o(y, a)$.

Thus, the above expression becomes:

$$\begin{aligned}
\Delta W & \approx \left(V^c(y, a) - V^s(y, a) \right) * d\mu^o(y, a) \\
& + \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') * \left(u_c(c, \chi h; p) * (1 + \kappa_b) \right) * d\mu^r(y', a') \right\} * \frac{\Delta p}{\Delta S} * \eta(\theta(y, a)) * d\mu^o(y, a) \\
& - \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \left(\sigma(y', a') * \eta(\theta(y', a')) * u_c(c, h; p) \right) * d\mu^o(y', a') \right\} \\
& \quad * \frac{\Delta p}{\Delta S} * \eta(\theta(y, a)) * d\mu^o(y, a)
\end{aligned}$$

Rewriting the expression above, one obtains:

$$\begin{aligned}
\Delta W & \approx \left[\left(V^c(y, a) - V^s(y, a) \right) \right. \\
& + \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') * \left(u_c(c, \chi h; p) * (1 + \kappa_b) \right) * d\mu^r(y', a') \right\} * \frac{\Delta p}{\Delta S} * \eta(\theta(y, a)) \\
& - \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \left(\sigma(y', a') * \eta(\theta(y', a')) * u_c(c, h; p) \right) * d\mu^o(y', a') \right\} * \frac{\Delta p}{\Delta S} * \eta(\theta(y, a)) \left. \right] \\
& \quad * d\mu^o(y, a)
\end{aligned}$$

As the Planner's optimal choice assigns all agents with state (y, a) to sell, the perturbation should not increase welfare. Further, if there were no change in welfare, then sale and repayment would both be optimal to the Planner for agents with state (y, a) . As we have assumed that sale is the unique optimal choice for the Planner for

agents with state (y, a) , the perturbation must yield strictly lower welfare. Hence, $\Delta W < 0$, and since $d\mu^o(y, a) > 0$, the expression in square brackets must be negative. Thus, the Planner chooses sale if the following condition holds:

$$\begin{aligned} \Delta W \approx & \left[V^s(y, a) - V^c(y, a) \right. \\ & - \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') * (u_c(c, \chi h; p) * (1 + \kappa_b)) * d\mu^r(y', a') \right\} * \frac{\Delta p}{\Delta S} * \eta(\theta(y, a)) \\ & \left. + \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} (\sigma(y', a') * \eta(\theta(y', a'))) * u_c(c, h; p) * d\mu^o(y', a') \right\} * \frac{\Delta p}{\Delta S} * \eta(\theta(y, a)) \right] \\ & > 0 \end{aligned}$$

Expanding the expression for $V^s(y, a)$ using equation (2.5) and factoring out $\eta(\theta(y, a))$ that is assumed to be positive, the Planner chooses sale if the following condition holds:

$$\begin{aligned} \Delta W \approx & \left[(v^s(y, a) - V^c(y, a)) \right. \\ & - \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') * (u_c(c, \chi h; p) * (1 + \kappa_b)) * d\mu^r(y', a') \right\} * \frac{\Delta p}{\Delta S} \\ & \left. + \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} (\sigma(y', a') * \eta(\theta(y', a'))) * u_c(c, h; p) * d\mu^o(y', a') \right\} * \frac{\Delta p}{\Delta S} \right] \\ & > 0 \end{aligned}$$

□

2.7.2.4 Sale choice with the default option

Proposition 3: The Planner chooses sale ($\sigma(y, a) = 1$) for an owner if the following condition holds:

$$V^s(y, a) - V^c(y, a) + \text{PE} + \text{Non-pecuniary externality} > 0 \quad (2.34)$$

Proof. The approach here is to use a perturbation argument in the spirit of Davila

et al. [2012]. Throughout the proof, I use the expression for the SWF in equation (2.32) augmented to include lender payoffs *ex post*:

$$\sum_{y \in Y} \int_{a \in \bar{A}} \sigma(y, a) * \left(1 - \eta(\theta(y, a))\right) * 1_{(\zeta p + a < 0)} * (\zeta p + a) * d\mu^o(y, a)$$

Let the Planner's sale choice be $\sigma(y, a)$. Consider a perturbation where the positive mass $d\mu^o(y, a)$ of owners with state (y, a) for whom the unique optimal choice is to sell are now switched to continuation status, i.e. they switch from $\sigma(y, a) = 1$ to $\sigma(y, a) = 0$. This reduces housing supply by an amount $\Delta S = \eta(\theta(y, a)) * d\mu^o(y, a)$, and I denote the resulting price change by $\Delta p \geq 0$. Finally, let the change in felicity of an agent due to a price change be denoted by $\Delta_p u(c, h; p)$.

The resulting change in social welfare from the perturbation, denoted by ΔW , is:

$$\begin{aligned} \Delta W = & \left(V^c(y, a) - V^s(y, a) + \left(1 - \eta(\theta(y, a))\right) * 1_{(\zeta p + a < 0)} * (\zeta p + a) \right) * d\mu^o(y, a) \\ & + \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') \left(\Delta_p u(c, \chi h; p) \right) * d\mu^r(y', a') \\ & + \sum_{y' \in Y} \int_{a' \in \bar{A}} \left(\sigma(y', a') * \eta(\theta(y', a')) * \Delta_p u(c, h; p) \right) * d\mu^o(y', a') \\ & + \sum_{y' \in Y} \int_{a' \in \bar{A}} \sigma(y', a') * \left(1 - \eta(\theta(y', a'))\right) * 1_{(\zeta p + a' < 0)} * (\zeta \Delta p) * d\mu^o(y', a') \end{aligned}$$

Using the first order approximation $u'(c)\Delta c \approx \Delta u$, consider for example the change in utility of sellers due to the price change. Seller consumption is $c = y + a + p(\theta) - \frac{a'}{R} - \rho$, so $\Delta c = \frac{\Delta p(\theta)}{\Delta p} * \Delta p$. Hence, $\Delta_p u(c, h; p) = u_c(c, h; p) * \frac{\Delta p(\theta)}{\Delta p} * \Delta p = -u_c(c, h; p) * \frac{\Delta p(\theta)}{\Delta p} * \frac{\Delta p}{\Delta S} * d\mu^o(y, a)$. Similarly, for buyers, the change in utility is $\Delta_p u(c, \chi h; p) = u_c(c, \chi h; p) * (1 + \kappa_b) * \frac{\Delta p}{\Delta S} * d\mu^o(y, a)$.

Thus, the above expression becomes:

$$\Delta W \approx \left(V^c(y, a) - V^s(y, a) + \left(1 - \eta(\theta(y, a))\right) * 1_{(\zeta p + a < 0)} * (\zeta p + a) \right) * d\mu^o(y, a)$$

$$\begin{aligned}
& + \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') * (u_c(c, \chi h; p) * (1 + \kappa_b)) * d\mu^r(y', a') \right\} * \frac{\Delta p}{\Delta S} * d\mu^o(y, a) \\
& - \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} (\sigma(y', a') * \eta(\theta(y', a')) * u_c(c, h; p)) * d\mu^o(y', a') \right\} \\
& \quad * \frac{\Delta p}{\Delta S} * d\mu^o(y, a) \\
& - \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \sigma(y', a') * (1 - \eta(\theta(y', a'))) * 1_{(\zeta p + a' < 0)} * \zeta * d\mu^o(y', a') \right\} \\
& \quad * \frac{\Delta p}{\Delta S} * \eta(\theta(y, a)) * d\mu^o(y, a)
\end{aligned}$$

Rewriting the expression above, one obtains:

$$\begin{aligned}
\Delta W \approx & \left[\left(V^c(y, a) - V^s(y, a) + (1 - \eta(\theta(y, a))) * 1_{(\zeta p + a < 0)} * (\zeta p + a) \right) \right. \\
& + \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') * (u_c(c, \chi h; p) * (1 + \kappa_b)) * d\mu^r(y', a') \right\} * \frac{\Delta p}{\Delta S} \\
& - \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} (\sigma(y', a') * \eta(\theta(y', a')) * u_c(c, h; p)) * d\mu^o(y', a') \right\} * \frac{\Delta p}{\Delta S} \\
& \left. - \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \sigma(y', a') * (1 - \eta(\theta(y', a'))) * 1_{(\zeta p + a' < 0)} * \zeta * d\mu^o(y', a') \right\} * \frac{\Delta p}{\Delta S} \right] \\
& * \eta(\theta(y, a)) * d\mu^o(y, a)
\end{aligned}$$

As the Planner's optimal choice assigns all agents with state (y, a) to sell, the perturbation should not increase welfare. Further, if there were no change in welfare, then sale and repayment would both be optimal to the Planner for agents with state (y, a) . As we have assumed that sale is the unique optimal choice for the Planner for agents with state (y, a) , the perturbation must yield strictly lower welfare. Hence, $\Delta W < 0$, and since $d\mu^o(y, a) > 0$, the expression in square brackets must be negative. Thus, the Planner chooses sale if the following condition holds:

$$\Delta W \approx \left[\left(V^s(y, a) - V^c(y, a) + (1 - \eta(\theta(y, a))) * 1_{(\zeta p + a < 0)} * (\zeta p + a) \right) \right]$$

$$\begin{aligned}
& - \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') * (u_c(c, \chi h; p) * (1 + \kappa_b)) * d\mu^r(y', a') \right\} * \frac{\Delta p}{\Delta S} \\
& + \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} (\sigma(y', a') * \eta(\theta(y', a'))) * u_c(c, h; p) * d\mu^o(y', a') \right\} * \frac{\Delta p}{\Delta S} \\
& + \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} \sigma(y', a') * (1 - \eta(\theta(y', a'))) * 1_{(\zeta p + a' < 0)} * \zeta * d\mu^o(y', a') \right\} * \eta(\theta(y, a)) * \frac{\Delta p}{\Delta S} \\
& > 0
\end{aligned}$$

□

2.7.2.5 Tightness choice

Proposition 4: The Planner's interior F.O.C for constrained efficient tightness choice is given by:

$$\eta'(\theta(y, a)) \left(v^s(y, a) - V^c(y, a) \right) + \eta(\theta(y, a)) v_p^s(y, a) p'(\theta) + PE_\theta = 0$$

where:

$$\begin{aligned}
PE_\theta = & \left\{ - \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') (u_c(c, \chi h)) * d\mu^r(y', a') \right. \\
& + \sum_{y' \in Y} \int_{a' \in \bar{A}} (\sigma(y', a') * \eta(\theta(y', a'))) * u_c(c, h) * \frac{dp(\theta(y', a'))}{dp} * d\mu^o(y', a') \left. \right\} \\
& * \frac{dp}{dS} \quad (2.35)
\end{aligned}$$

Proof. I use a perturbation argument similar to the proof of Propositions 2 and 3. Throughout the proof, I use the expression for the SWF in equation (2.32).

Let the Planner's tightness choice be $\theta(y, a)$. Consider a perturbation where the Planner increases tightness choice to $\theta(y, a) + \epsilon$, where $\epsilon > 0$. This increases sale probability by $\eta'(\theta(y, a)) * d\theta(y, a)$, which in turn increases housing supply by $dS = \eta'(\theta(y, a)) * d\theta(y, a) * d\mu^o(y, a)$. I denote the resulting price change by $dp \leq 0$.

The change in welfare from this perturbation, denoted by dW , is:

$$\begin{aligned}
 dW = & \left\{ \eta'(\theta(y, a)) \left(v^s(y, a) - V^c(y, a) \right) + \eta(\theta(y, a)) v_p^s(y, a) p'(\theta) \right\} * d\theta(y, a) * d\mu^o(y, a) \\
 & + \sum_{y' \in Y} \int_{a' \in \bar{A}} \omega(y', a') * \left(d_\theta u(c, \chi h) \right) * d\mu^r(y', a') \\
 & + \sum_{y' \in Y} \int_{a' \in \bar{A}} \left(\sigma(y', a') * \eta(\theta(y', a')) * d_\theta u(c, h) \right) * d\mu^o(y', a')
 \end{aligned}$$

Using $u'(c)dc = du$, consider for example the change in utility of sellers due to the price change. Seller consumption is $c = y + a + p(\theta) - \frac{a'}{R} - \rho$, so $dc = \frac{dp(\theta)}{dp} * dp$. Hence, $du(c, h; p) = u_c(c, h; p) * \frac{dp(\theta)}{dp} * dp = u'(c, h; p) * \frac{dp(\theta)}{dp} * \frac{dp}{dS} * \eta'(\theta(y, a)) * d\mu^o(y, a)$. Similarly, for buyers, the change in utility from a price change is $du(c, \chi h; p) = u_c(c, \chi h; p) * (1 + \kappa_b) * \frac{dp}{dS} * \eta'(\theta(y, a)) * d\mu^o(y, a)$.

Thus, the above expression becomes:

$$\begin{aligned}
 dW = & \left\{ \eta'(\theta(y, a)) \left(v^s(y, a) - V^c(y, a) \right) + \eta(\theta(y, a)) v_p^s(y, a) p'(\theta) \right\} * d\theta(y, a) * d\mu^o(y, a) \\
 & \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} -\omega(y', a') * \left(u_c(c, \chi h) \right) * d\mu^r(y', a') \right. \\
 & \left. + \sum_{y' \in Y} \int_{a' \in \bar{A}} \left(\sigma(y', a') * \eta(\theta(y', a')) * u_c(c, h) \right) * \frac{dp(\theta(y', a'))}{dp} * d\mu^o(y', a') \right\} \\
 & * d\theta(y, a) * d\mu^o(y, a) * \frac{dp}{dS} * \eta'(\theta(y, a))
 \end{aligned}$$

As $\theta(y, a)$ is the unique optimal tightness choice for the Planner, the perturbation should not affect welfare. If $dW < 0$, then lowering tightness could raise welfare. On the other hand, if $dW > 0$, then raising tightness could increase welfare. Hence, it must be the case that $dW = 0$. If the constraint $\theta \geq 0$ is binding with Lagrange multiplier $\lambda(y, a)$, this condition becomes $dW = -\lambda(y, a)$.

Hence, the Planner's first order condition for tightness choice is:

$$\begin{aligned}
& \left\{ \eta'(\theta(y, a)) \left(v^s(y, a) - V^c(y, a) \right) + \eta(\theta(y, a)) v_p^s(y, a) p'(\theta) \right\} * d\theta(y, a) * d\mu^o(y, a) \\
& \quad \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} -\omega(y', a') * (u_c(c, \chi h)) * d\mu^r(y', a') \right. \\
& \quad \left. + \sum_{y' \in Y} \int_{a' \in \bar{A}} \left(\sigma(y', a') * \eta(\theta(y', a')) * u_c(c, h) \right) * \frac{dp(\theta(y', a'))}{dp} * d\mu^o(y', a') \right\} \\
& \quad * d\theta(y, a) * d\mu^o(y, a) * \frac{dp}{dS} * \eta'(\theta(y, a)) \leq 0
\end{aligned}$$

With positive density $d\mu^o(y, a)$, positive tightness choice $\theta(y, a)$ and since $d\theta(y, a) = \epsilon > 0$, this can be written as:

$$\begin{aligned}
& \left\{ \left(v^s(y, a) - V^c(y, a) \right) + \frac{\eta(\theta(y, a))}{\eta'(\theta(y, a))} v_p^s(y, a) p'(\theta) \right\} \\
& \quad \left\{ \sum_{y' \in Y} \int_{a' \in \bar{A}} -\omega(y', a') * (u_c(c, \chi h)) * d\mu^r(y', a') \right. \\
& \quad \left. + \sum_{y' \in Y} \int_{a' \in \bar{A}} \left(\sigma(y', a') * \eta(\theta(y', a')) * u_c(c, h) \right) * \frac{dp(\theta(y', a'))}{dp} * d\mu^o(y', a') \right\} \\
& \quad * \frac{dp}{dS} \leq 0
\end{aligned}$$

which is the condition in equations (2.30) – (2.31).

□

Chapter 3

Downward nominal wage rigidity, limited monetary policy responsiveness and regional labour mobility

3.1 Introduction

Rural-urban migration is an important feature of the development process that has been studied through various prisms: the long run structural transformation of an economy from predominantly agrarian to manufacturing and service sector dominated; and a shorter run focus on rural-urban wage differentials, urban unemployment and regional labour sorting.

While the process of urbanization and structural transformation is well known (see e.g. [Herrendorf et al. \[2014\]](#)), the tendency to urbanize has been influenced through the use of different policies affecting labour mobility. A prominent example is the *hukou* (household registration) system in China instituted in the late 1950s, which aimed to restrict rural-urban migration as part of an urban industrialization strategy that extracted rural agrarian surpluses and hence required the rural population to be tied to the land ([Chan \[2010\]](#), [Naughton \[2018\]](#)). More generally, the efficiency gains from differently skilled individuals moving to (typically urban) locations where they are more productive are countered by the ensuing congestion costs, urban unemployment

and visible urban distress (Lall et al. [2006]).

The influential analysis by Harris and Todaro [1970] showed that urban job creation policies could potentially lead to higher urban unemployment through induced migration. This argument, known as the Todaro paradox, could explain both restrictive migration policies and an emphasis on rural development programs in several developing countries. However, the Todaro paradox has not been well established empirically. Instead, Lall et al. [2006] highlight the beneficial effects of migration on rural outcomes, through remittances that boost rural capital investment and reduce poverty. A more fundamental objection to policies restricting migration is that they limit the right of migrants to move to better their lives.

A recent literature has also documented the presence of rural-urban real wage gaps for individuals using longitudinal data that are not entirely explained away by sorting on unobserved abilities (see e.g. Lagakos [2020]). Financial frictions might prevent the movement of labour to sort efficiently and close these wage gaps. This has been used to motivate migration subsidies in the developing country context (e.g. Bryan et al. [2014], Lagakos et al. [2018]).

This paper contributes to the literature on migration policies and their beneficial impact on outcomes in the originating (rural) region. It studies migration following adverse rural labour demand shocks. It applies the static model of labour mobility in currency unions developed by Farhi and Werning [2014] to a two region (rural-urban) economy with downward rural nominal wage rigidity (DNWR) and limited monetary policy responsiveness. In particular, monetary policy does not stabilize the *aggregate* economy following asymmetric regional shocks. I show that a social planner subject to these institutional and policy frictions but with the power to relocate individuals would generally choose a location decision that differs from private agents' location decisions. This discrepancy arises due to an aggregate demand externality. Unlike individuals who migrate based on utility differentials, the social planner accounts for the impact of relocation on the welfare of all individuals. The inefficiency wedge arises due to the binding downward nominal wage rigidity in rural areas and the limited responsiveness of monetary policy to negative rural labour demand shocks. Under plausible conditions, the planner would choose to encourage rural-urban migration.

This inefficiency result was first derived by Farhi and Werning [2014]¹. This paper

¹Lagakos et al. [2018] discuss constrained inefficient migration arising due to a pecuniary externality:

builds on their article in the following respects. *First*, I show that the aggregate demand externality from migration is stronger than in their setting. Not only does the emigration of rural workers lead to higher per-capita allocations for the rural workers who remain behind, but aggregate rural output also increases with urban population size in the presence of demand linkages. I also generalize the expression for the migration wedge to account for diminishing marginal product of labour in production.

Second, I discuss when migration to the urban region might actually worsen rural outcomes. I show that only strong negative externalities (for e.g. due to congestion) in the urban region from migration could possibly make rural residents who do not migrate worse off.

Third, I also relate the model to the development literature on internal migration. The model environment features limited monetary policy responsiveness, regional demand linkages and a dependence of aggregate urban output on urban population size. Hence, one can evaluate how rural output is stimulated by a policy encouraging migration. I relate the migration based rural output multiplier to rural output multipliers associated with transfers to rural residents that restrict labour mobility in a manner described below. I also incorporate heterogeneously skilled labour and factor input demand linkages and show that these tend to amplify the migration based rural output multiplier.

3.1.1 Literature review

This paper is related to the literature on the benefits of labour mobility in currency unions advanced recently by [Farhi and Werning \[2014\]](#). They develop theoretically the insight of [Mundell \[1961\]](#) that migration boosts depressed regions (the origin) and cools down booming regions (the destination). Quantitative analyses of the role of labour mobility in fostering macroeconomic stabilization in currency unions include [House et al. \[2018\]](#), [Hauser and Seneca \[2019\]](#), where unemployment arises in a frictional labour market. [Howard \[2020\]](#) incorporates a housing sector in addition to labour mobility to motivate his finding that the cooling down effect does not hold in the United States.

rural out-migration raises rural wages and imperfect risk-sharing leads to a distributive externality associated with individual migration choice, in the terminology of [Dávila and Korinek \[2018\]](#).

The paper is also related to the literature focusing on downward nominal wage rigidity, monetary policy limitations and involuntary unemployment (e.g. [Schmitt-Grohé and Uribe \[2016\]](#), [Shen and Yang \[2018\]](#)). [Kaur \[2019\]](#) and [Breza et al. \[2021\]](#) provide recent evidence on rural DNWR and labour rationing from India. [Guerrieri et al. \[2020\]](#) consider wage rigidity, limited monetary policy responsiveness and the effect of negative supply shocks such as the Covid-19 pandemic.

[Lall et al. \[2006\]](#) is a comprehensive review of the theoretical and empirical literature on rural-urban migration in developing countries, with an emphasis on policy implications. Although the model below is static, it shares many features with recent structural and quantitative articles focusing on the productivity and welfare effects of migration (see e.g. [Lagakos et al. \[2018\]](#), [Hnatkovska and Lahiri \[2018\]](#) and [Bryan and Morten \[2019\]](#)). Quantitative spatial models also feature agent heterogeneity, goods trade and labour mobility; they have been used to quantify the welfare impact of migration restrictions (see e.g. [Desmet et al. \[2018\]](#), [Redding and Rossi-Hansberg \[2017\]](#)).

Finally, the transfer based rural output multiplier is related to the extensive literature on fiscal multipliers summarized in, e.g. [Farhi and Werning \[2016\]](#) and [Chodorow-Reich \[2019\]](#). [Lagakos et al. \[2018\]](#) compare a policy fostering migration to rural workfare and transfer programs in a quantitative model.

3.1.2 Outline of paper

Section [3.2](#) describes the model and the competitive equilibrium. Section [3.3](#) describes the social planner's problem and the constrained efficient labour mobility choice. It also discusses the conditions under which rural-urban labour mobility might be inefficiently high. Section [3.4](#) relates the rural output multiplier from relocation to that arising from regional transfers. Section [3.5](#) numerically illustrates the magnitude of the migration wedge and multipliers. Section [3.6](#) concludes. Supplementary material is contained in the appendices.

3.2 A two-region model with demand linkages and labour mobility

The analysis employs the static model of labour mobility in a currency union described in [Farhi and Werning \[2014\]](#), specifically their model of external demand imbalances. There are two regions, u and r , that shall henceforth refer to the 'urban' and 'rural' region respectively. These regions comprise the entire union (country). Each region produces a good using the labour of its residents, which is consumed by the residents of both regions. Labour is mobile between the two regions.

The source of inefficiency in the setup is that rural nominal wages are rigid downward, based on the evidence in [Kaur \[2019\]](#). Adverse shocks to labour demand in the rural region, such as negative rainfall shocks, would induce involuntary rural unemployment and an adjustment in the relative price of goods produced in the two regions (the terms of trade). Monetary policy could respond to rural labour demand shocks by adjusting the price of the urban good so as to replicate the full employment allocation, but in this paper I assume that monetary policy does not (or cannot) replicate the first-best allocation. This generates a wedge between the equilibrium and first-best allocations that results in an inefficiency associated with individual mobility choice.

3.2.1 Environment

Agents

There is a continuum of agents with finite number of types $j \in J$ of mass μ^j . These agents reside in either region, and I denote the mass of agents of type j in region i by μ_{ij} . Then, the total mass of agents residing in region i is $\mu^i = \sum_j \mu_{ij}$. The total mass of agents in the economy is $\sum_i \mu^i = 1$.

Agents of type j residing in region i derive utility from the consumption of goods produced in the rural (C_{rj}^i) and the urban regions (C_{uj}^i), and face a utility cost from labour supply, N_{ij} . I assume that in any region $i \in \{r, u\}$, the utility function U^{ij} represents the same preference ordering for all agents of type j , hence can be expressed instead as $U^i(C_j^i, N_{ij})$.

The utility function for most of the analysis shall take the GHH form ([Greenwood](#)

et al. [1988]) in order to rule out wealth effects on labour supply². This specification is motivated by, for e.g., Schmitt-Grohé and Uribe [2012], Shen and Yang [2018]:

$$U^i(C_{rj}^i, C_{uj}^i, N_{ij}) = \log(C_j^i - \frac{N_{ij}^{1+\phi}}{1+\phi}) \quad (3.1)$$

In the above, C_j^i is a Cobb-Douglas aggregator over agent j resident in region i 's consumption of the rural and urban goods, with expenditure share $1 - \gamma_i$ on the local good. Note that, due to the homotheticity of preferences over consumption goods and the assumptions about preferences, the expenditure shares depend only on location and not on type. For instance, for a rural resident of type j , we have:

$$C_j^r = \frac{(C_{rj}^r)^{1-\gamma_r} (C_{uj}^r)^{\gamma_r}}{(1 - \gamma_r)^{1-\gamma_r} (\gamma_r)^{\gamma_r}} \quad (3.2)$$

Agents of type j residing in region i maximize utility subject to the following budget constraint:

$$P_r C_{rj}^i + P_u C_{uj}^i \leq W_i N_{ij} + T_i \quad (3.3)$$

Here, P_r and P_u are the prices of the rural and urban goods respectively, W_i is the wage in region i and T_i is a lump sum transfer to residents of region i .

I divide the utility maximization problem of an agent of type j residing in region i into two parts. First, given overall consumption expenditure $P^i C_j^i$, the consumption of the domestic good for the Cobb-Douglas aggregator in equation (3.2) is:

$$C_{ij}^i = \frac{(1 - \gamma_i) P^i C_j^i}{P_i} \quad (3.4)$$

This yields the following price index P^i in region i (with $i, k \in \{r, u\}$):

$$P^i = P_i^{1-\gamma_i} P_k^{\gamma_i} \quad (3.5)$$

Next, consider the choice of overall consumption and labour supply. Combining the

²Appendix 3.7.2 shows that the results hold for separable preferences too.

first-order conditions for aggregate consumption and labour supply, we have:

$$\frac{-U_{N_{ij}^s}}{U_{C_j^i}} = \frac{W_i}{P_i} \quad (3.6)$$

With GHH preferences, this is simply:

$$(N_{ij}^s)^\phi = \frac{W_i}{P_i} \quad (3.7)$$

As equations (3.6) and (3.7) indicate, all workers in region i would choose to work the same amount of hours irrespective of type. The superscript s refers to the *desired* labour supply of workers in region i ; in the presence of downward nominal wage rigidity, this could differ from actual labour supplied.

Downward nominal wage rigidity in the rural region

In this static model, I assume that nominal wages are flexible in the urban region and are rigid downwards in the rural region. Hence, nominal urban wages can adjust downward to accommodate an influx of rural migrants.

While Kaur [2019] documents DNWR in rural labour markets in India, the evidence on the flexibility of urban wages in developing countries is sparse and often indirect. Freeman [2010] reviews the empirical literature on minimum wage policies and their effects on employment and finds mixed evidence, with the negative effects generally being small. Further, unemployment tends to exert downward pressure on urban wages, contrary to the positive wage-unemployment relationship implied by the Harris-Todaro model (Harris and Todaro [1970]).

The assumptions about wage rigidity in rural and urban sectors are meant to capture very broadly the idea that rural migrants can find *some* source of urban employment, possibly less-skill intensive urban jobs in the informal or construction sector, as is common in many developing countries³. Indeed, the presence of formal labour market restrictions such as minimum wages might drive employers to hire from the informal sector where nominal wages are more flexible.

³Alternatively, one could assume that the downward nominal wage rigidity constraint in the urban region is less stringent.

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This contrasts with the treatment in [Harris and Todaro \[1970\]](#), where urban wages are fixed and urban unemployment equilibrates rural-urban migratory flows. The intention in this paper is to represent in a simple manner a scenario with a lack of alternative rural employment opportunities and the possibility of employment in the urban region.

The assumption about downward nominal wage rigidity (DNWR) in the rural region follows from the findings in [Kaur \[2019\]](#), and takes the simple form following [Schmitt-Grohé and Uribe \[2016\]](#):

$$W_r \geq \bar{W} \quad (3.8)$$

Following negative labour demand shocks in the rural region that lower the market clearing rural wage below \bar{W} , the DNWR constraint on rural wages binds and there is involuntary unemployment:

$$u_r = 1 - \frac{N_r}{N_r^s} \quad (3.9)$$

Nominal rural wages and rural labour supply satisfy the following slackness condition:

$$(N_r^s - N_r)(W_r - \bar{W}) = 0 \quad (3.10)$$

In the urban region, desired and actual labour supply by workers coincide, i.e. $N_u^s = N_u$.

Labour mobility decision

Finally, agents can move from region i to region k after incurring some utility cost ρ_{ik} . An agent of type j residing in region i would choose to move to region k if he were better off in terms of utility from doing so. If this were the case, no resident of type j would choose to reside in region i , leading to the following:

$$\mu_{ij} = 0 \quad \text{if} \quad U^i(C_j^i, N_i) < U^k(C_j^k, N_k) - \rho_{ik}^j \quad (3.11)$$

CHAPTER 3. WAGE RIGIDITY, MONETARY POLICY AND REGIONAL LABOUR MOBILITY

Firms

Goods in each region are produced by a representative, price-taking firm using labor according to the production function:

$$Y_i = A_i(N^i)^{1-\alpha} \quad (3.12)$$

The representative firm chooses labour demand in order to maximize after-tax profits:

$$\Pi^i = (1 - \tau_{\pi,i})[P_i Y_i - W_i N_i] \quad (3.13)$$

Labour demand in region i is then given by:

$$\frac{(1 - \alpha)P_i A_i}{W_i} = (N^i)^\alpha \quad (3.14)$$

Government

The government is assumed to balance the budget in each region. I assume that regional firm profits are completely taxed away ($\tau_{\pi,i} = 1$) and distributed as transfers to residents of that region:

$$\begin{aligned} \sum_j \mu_{ij} T_i &= P_i Y_i - W_i N_i \\ \implies T_i &= \frac{P_i Y_i - W_i N_i}{\mu_i} \end{aligned} \quad (3.15)$$

This is equivalent to a case where there are no taxes or transfers, and the profits of a regional firm owned by that region's residents are divided equally among its owners.

Monetary policy

Monetary policy is assumed to control the price of the urban good P_u ⁴. Let p denote the relative price of urban and rural goods:

$$p = \frac{P_u}{P_r} \quad (3.16)$$

When the rural DNWR constraint does not bind, p is determined entirely by relative productivity shocks $\{A_i\}_{i \in \{r,u\}}$ and the population shares μ^i in rural and urban regions. This, along with the monetary policy-set P_u , helps determine P_r .

However, when the rural DNWR constraint does bind, the equilibrium allocation will depend on monetary policy through the choice of P_u . I assume that monetary policy sets P_u so as to keep the relative price p unchanged at the initial, pre-rural labour demand shock level. Hence the monetary policy response is *limited* and cannot achieve the rural 'full employment' allocation where $N_r^s = N_r$ ⁵.

I do not consider here why the monetary policymaker does not (or cannot) act to stabilize the aggregate economy⁶; instead I shall work out the consequences under such a scenario. This implies that the ensuing analysis is *dependent* on the assumed monetary policy: Farhi and Werning [2014] call the planning analysis under this scenario a 'restricted' planning problem. I will henceforth treat this dependence as implicit.

Denote the initial relative price by p_0 . The rural goods price P_r adjusts to satisfy the goods market clearing condition discussed below, and the monetary policymaker in turn sets P_u to keep the relative price at p_0 .

⁴In a dynamic model, one could assume that the urban sector features price stickiness and monopolistic competition as in a standard New Keynesian model and monetary policy is set using an interest rate rule (e.g. Aoki [2001]).

⁵Although my focus is on migration choices when the monetary policy response is limited, optimal monetary policy would set P_u to maximize total welfare, following Farhi and Werning [2014]. One can show that the F.O.C for P_u is analogous to the F.O.C for migration choice derived below in equation (3.30).

⁶In an open economy model where urban sector firms face collateral constraints (Cavallino and Sandri [2020]), monetary policy might face an effective lower bound on interest rates that would hinder stabilization following asymmetric regional shocks. Sah and Stiglitz [1984] discuss a case where the policymaker might distort the terms of trade to favour the urban industrialized sector.

3.2.2 Equilibrium

Given monetary policy that determines P_u as discussed above, an equilibrium **without free mobility** is a set of masses μ_{ij} , nominal wages W_i , rural goods price P_r , consumption of rural and urban goods C_{rj}^i and C_{uj}^i respectively, labour demand N^i and output Y_i , profits Π^i and transfers T_i ; $\forall i \in \{r, u\}$, $j \in J$, satisfying equations (3.4) – (3.6), (3.8) – (3.10), (3.11), (3.14), the condition: $\mu^i = \sum_j \mu_{ij}$ and the market clearing conditions:

$$\sum_{i,j} \mu_{ij} C_{kj}^i = Y_k \quad \forall k \in r, u \quad (3.17)$$

$$\sum_j \mu_{ij} N_{ij} = N^i \quad (3.18)$$

An equilibrium *with free mobility* also requires equation (3.11) to hold.

I now derive individual consumption and labour supply choices. As noted above, all individuals of type j in region i supply the same amount of labour, $N_{ij} = N_i$, such that equation (3.18) becomes:

$$N_i = \frac{N^i}{\mu^i} \quad (3.19)$$

Combining equations (3.3), (3.4) and (3.15), one obtains:

$$P^i C_j^i = W_i N_{ij} + \frac{P_i Y_i - W_i N^i}{\mu_i} = \frac{P_i Y_i}{\mu_i} \quad (3.20)$$

This yields, using equation (3.4), the consumption of good k by the resident of type j in region i :

$$C_k^i = C_{kj}^i = \frac{\gamma_i P_i Y_i}{\mu^i P_k} \quad (3.21)$$

This is again, owing to the assumption made about preferences, independent of agent type.

Finally, I simplify the market clearing condition in equation (3.17) using $C_{kj}^i = C_k^i$ and equation (3.21):

$$\gamma_k P_k Y_k = \gamma_i P_i Y_i \quad (3.22)$$

3.2.2.1 Rural labour demand shocks and a binding rural DNWR constraint

Consider now a adverse shock to A_r , arising for instance due to a negative rainfall shock. This would lower rural labour demand, from equation (3.14), but would also lower the rural market clearing wage.

I consider the case where the rural market clearing wage is below the wage floor in equation (3.8), implying from equation (3.10) that there is involuntary unemployment:

$$N_r < N_s^r = \left(\frac{\bar{W}}{P_r}\right)^{\frac{1}{\phi}}$$

Actual labour supplied is determined by labour demand. From equation (3.14), aggregate rural labour demand at rural wage \bar{W} is:

$$N^r = \left(\frac{(1-\alpha)P_r A_r}{\bar{W}}\right)^{\frac{1}{\alpha}} \quad (3.23)$$

As all rural residents irrespective of type supply the same amount of labour, we have $N_r = \frac{N^r}{\mu^r}$.

In the urban sector, there is no involuntary unemployment and urban labour supply is obtained from equations (3.7) and (3.14):

$$N_u^\phi = \frac{(1-\alpha)P_u A_u (\mu^u N_u)^{-\alpha}}{P^u} \quad (3.24)$$

From equations (3.5) and (3.16):

$$\frac{P_u}{P^u} = \frac{P_u}{P_u^{1-\gamma_u} P_r^{\gamma_u}} = p^{\gamma_u} \quad (3.25)$$

Hence, urban labour supply is:

$$N_u = \left((1-\alpha)p^{\gamma_u} A_u (\mu^u)^{-\alpha}\right)^{\frac{1}{\alpha+\phi}} \quad (3.26)$$

As discussed above, monetary policy sets $p = p_0$. This determines N_u and one can then determine P_r and N_r using equations (3.22) – (3.23).⁷

⁷The equilibrium allocation is derived in appendix 3.7.1.

Labour wedge

The *labour wedge* in region i is given by:

$$\tau_i = 1 + \frac{U_{N_i} P_i}{U_{C^i} A_i P_i (1 - \alpha) (N^i)^{-\alpha}} \quad (3.27)$$

In the first-best allocation, this is zero in both regions. However, when the monetary policy response is limited and the rural DNWR constraint binds, then τ_r is positive as there is involuntary rural unemployment.

3.3 Social planner's choice of labour mobility

3.3.1 Social planner's problem

The social welfare function is defined as the weighted average of agents' utilities, with the positive Pareto weight for an agent of type j denoted by λ^j . The Social planner maximizes this social welfare function over the set of equilibria **without** free mobility. In other words, the planner has the power to relocate individuals across regions, but the planner is also subject to the utility cost ρ_{ik}^j associated with moving a resident of type j in region i to region $k \neq i$, with $\rho_{ii}^j = 0 \forall j$.

The objective is to compare the constrained efficient labour mobility choices with individual labour mobility decisions given by equation (3.11). To this end, the planner chooses μ_{uj} , the mass of residents of type j in the urban region u , in order to maximize the following objective:

$$W = \mu_{uj} \lambda^j (U^u(C_j^u, N_{uj}) - \rho_{ru}^j) + (\mu^j - \mu_{uj}) \lambda^j U^r(C_j^r, N_{rj}) + \sum_i \sum_{j' \neq j} \lambda^{j'} \mu_{ij'} U^i(C_{j'}^i, N_{ij'}) \quad (3.28)$$

where we use the adding-up constraints $\forall j$:

$$\sum_i \mu_{ij} = \mu^j$$

and using $\forall i$:

$$\sum_j \mu_{ij} = \mu^i$$

The planner is also subject to the utility cost incurred when shifting agents from the rural to the urban region, which is associated with an increase in μ_{uj} .

We can use equations (3.12), (3.19), (3.20) to rewrite the utility for agent i as:

$$U^i(C_j^i, N_{ij}) = U^i\left(\frac{P_i A_i (N^i)^{(1-\alpha)}}{P^i \mu^i}, \frac{N^i}{\mu^i}\right) \quad (3.29)$$

Suppose the planner increases the mass of type j residents in region i by $d\mu_{ij}$. Using equation (3.29), the change in welfare is:

$$\begin{aligned} dW = & \left\{ \lambda^j \left(U^u(C_j^u, N_{uj}) - U^r(C_j^r, N_{rj}) - \rho_{ru}^j \right) \right. \\ & + \sum_{\hat{j}} \mu_{u\hat{j}} \lambda^{\hat{j}} \left[U_c^u \frac{(1-\alpha) P_u A_u (N^u)^{-\alpha}}{P^u} \frac{\partial \left(\frac{N^u}{\mu^u} \right)}{\partial \mu^u} \tau_u - \alpha U_c^u \frac{C^u}{\mu^u} \right] \\ & \left. + \sum_{\hat{j}} \mu_{r\hat{j}} \lambda^{\hat{j}} \left[U_c^r \frac{(1-\alpha) P_r A_r (N^r)^{-\alpha}}{P^r} \frac{\partial \left(\frac{N^r}{\mu^r} \right)}{\partial \mu^u} \tau_r + \alpha U_c^r \frac{C^r}{\mu^r} \right] \right\} d\mu_{uj} \quad (3.30) \end{aligned}$$

The planner would choose to increase the mass of type j residents in region i if the expression in braces in equation (3.30) is positive, i.e. if welfare increases from such a relocation.

Constrained inefficiency of individual migration choices

Comparing the private migration decision for an agent of type j moving from region r to region u in equations (3.11) to (3.30) normalized by the positive λ^j , we note that the constrained efficient relocation decision generally differs from the corresponding private migration decision. In other words, the private migration decision is generally constrained inefficient. The difference between the expressions in equations (3.11) and (3.30) is what I shall term the *migration wedge*, which I discuss next.

3.3.2 The migration wedge

The wedge between the constrained inefficient and private migration choice comprises the two terms in square brackets in equation (3.30):

$$\begin{aligned}\Omega_u = & \sum_{\hat{j}} \mu_{u\hat{j}} \lambda^{\hat{j}} \left[U_c^u \frac{(1-\alpha)P_u A_u (N^u)^{-\alpha}}{P^u} \frac{\partial \left(\frac{N^u}{\mu^u} \right)}{\partial \mu^u} \tau_u - \alpha U_c^u \frac{C^u}{\mu^u} \right] \\ & + \sum_{\hat{j}} \mu_{r\hat{j}} \lambda^{\hat{j}} \left[U_c^r \frac{(1-\alpha)P_r A_r (N^r)^{-\alpha}}{P^r} \frac{\partial \left(\frac{N^r}{\mu^r} \right)}{\partial \mu^u} \tau_r + \alpha U_c^r \frac{C^r}{\mu^r} \right] \quad (3.31)\end{aligned}$$

I now discuss how each component affects welfare following the change in μ_{uj} .

The first wedge component

Consider now the first term in each expression in the square bracket, written below for region i :

$$U_c^k \frac{(1-\alpha)P_k A_k (N^k)^{-\alpha}}{P^k} \frac{\partial \left(\frac{N^k}{\mu^k} \right)}{\partial \mu^i} \tau_k$$

This term arises as changing the share of workers of type j in region i affects μ^i and thereby affects the consumption and labour supply choices of agents in both regions. In particular, increasing μ^u negatively affects the labour supply and the per-capita consumption allocation in region u . In contrast, in this setting, increasing μ^u reduces μ^r and also affects N^r due to demand linkages.

This term depends on the labour wedge in region k and the variation of individual labour supply (N_k) with μ^i . As discussed above, the labour wedge in the urban region $\tau_u = 0$ but the labour wedge in the rural region is positive when the rural DNWR constraint binds. Hence, the first term in the square brackets is zero for the urban region but could be positive or negative for the rural region, depending on the sign of $\frac{\partial \left(\frac{N^r}{\mu^r} \right)}{\partial \mu^u}$. In our specification, N^r is increasing in μ^u and $\mu^r = 1 - \mu^u$, hence this partial derivative term is also positive.

The second wedge component

The overall sign of the wedge then depends on the other terms in the square brackets:

$$-\sum_{\hat{j}} \mu_{u\hat{j}} \lambda^{\hat{j}} \alpha U_c^u \frac{C^u}{\mu^u} + \sum_{\hat{j}} \mu_{r\hat{j}} \lambda^{\hat{j}} \alpha U_c^r \frac{C^r}{\mu^r}$$

These terms arise due to the assumption that the production function exhibits decreasing returns to scale. When the production function is linear ($\alpha = 0$), this term vanishes.

As seen in equation (3.20), regional consumption does not depend on agent type \hat{j} . Hence, the above equation can be written as:

$$\alpha U_c^r \frac{C^r}{\mu^r} \left(\sum_{\hat{j}} \mu_{r\hat{j}} \lambda^{\hat{j}} \right) \left\{ 1 - \frac{\gamma_u}{\gamma_r} \frac{U_c^u}{U_c^r} \frac{(\mu^r)^2}{(\mu^u)^2} \frac{P^r}{P^u} \frac{\sum_{\hat{j}} \mu_{u\hat{j}} \lambda^{\hat{j}}}{\sum_{\hat{j}} \mu_{r\hat{j}} \lambda^{\hat{j}}} \right\}$$

The sign of this expression depends on the sign of the term in braces. Suppose there is no home bias in either region (i.e. if $\gamma_r = \gamma_u = \frac{1}{2}$). This implies that $\frac{P^r}{P^u} = p^{1-2\gamma} = 1$.

Furthermore, the product $\frac{(\mu^r)^2}{(\mu^u)^2} \frac{\sum_{\hat{j}} \mu_{u\hat{j}} \lambda^{\hat{j}}}{\sum_{\hat{j}} \mu_{r\hat{j}} \lambda^{\hat{j}}}$ is dependent on the ratio $\frac{\mu_r}{\mu_u}$. Hence, if the ratio of rural to urban population is not too large⁸, and if rural agents have higher marginal utilities of consumption following an adverse rural labour demand shock, then the term in braces is likely to be positive.

I summarize the above discussion in the following proposition.

Proposition 1: Constrained efficient migration choices given monetary policy are generally inconsistent with free mobility. The migration wedge in equation (3.31) depends on the rural labour wedge and is positive if rural consumers are relatively more constrained than urban consumers and the rural population share is not too large.

Implementing the constrained efficient migration choice

The recent development literature on migration policies, such as Bryan et al. [2014], Lagakos et al. [2018], has relied on experiments offering conditional cash transfers to

⁸This is plausible given the rapid urbanization that characterizes the development process.

potential rural migrants. One can consider a similar lump-sum transfer policy in our setting to implement the constrained efficient migration decision.

The conditional transfer policy boosts urban income conditional on migration by s_{ru} such that:

$$U^u\left(\frac{P_u Y_u}{P^u \mu^u} + s_{ru}, N_u\right) = U^u\left(\frac{P_u Y_u}{P^u \mu^u}, N_u\right) + \Omega_u \quad (3.32)$$

In the above, Ω_u is the migration wedge in equation (3.31). Clearly, such a policy would imply that the migration decision of an agent of type j would coincide with the constrained efficient migration choice of the planner on behalf of this agent.

3.3.2.1 Comparison to the inefficiency results in Farhi and Werning [2014]

The above proposition is qualitatively similar to Proposition 9 in Farhi and Werning [2014] about the general constrained inefficiency of private labour mobility decisions. However, there are some differences that I now highlight.

The production function in Farhi and Werning [2014] is linear, which accounts for the omission of the second wedge component in their paper. Indeed, if one were to apply their framework where N^u and N^r do not depend on μ^u , one would obtain the overall migration wedge in equation (3.31). In the current environment, setting $\alpha = 0$ implies that N^u and hence N^r also depends *linearly* on μ^u . Therefore, a linear production function applied here also leads to constrained inefficiency in labour mobility choice.

Furthermore, in the environment described in this paper, with $\alpha > 0$, the labour supplied $\{N_u, N_r\}$ depend on μ^u as well. Hence, the relevant partial derivative in equation (3.31) depends on how *individual* labour supply in region k depends on an increase in μ^u .

In the rural region then, the first wedge component includes $\frac{\partial\left(\frac{N^r}{\mu^r}\right)}{\partial\mu^u}$. As μ^r and N^r are increasing in μ^u (the latter due to positive demand linkages), the first wedge component in the rural region is thereby *greater* than if N^r were unaffected by μ^u , which is the case discussed by Farhi and Werning [2014].

3.3.3 When is rural-urban migration inefficiently high?

As the preceding discussion and proposition have indicated, under plausible conditions the migration wedge is *positive*: rural-urban migration is inefficiently *low*.

Assuming that the second wedge component is net positive, it is interesting to consider whether this conclusion can be overturned. This would be consistent with policies that either explicitly restrict internal migration in several developing countries, or that choose to employ alternative policies to handle asymmetric regional shocks.

In the current framework, this would depend on the rural wedge component and in particular on how rural labour supply responds to greater migration to the urban region: $\frac{\partial \left(\frac{N^r}{\mu^r} \right)}{\partial \mu^u}$ in equation (3.31).

For the migration wedge to oppose rural-urban migration, one would require $\frac{N^r}{\mu^r}$ to be *decreasing* in μ^u . As $\mu^r = 1 - \mu^u$ is decreasing in urban population, this requires N^r to be decreasing in μ^u sufficiently so as to overcome the change in μ^r .

The dependence of rural employment and output on the urban population is derived using equation (3.22), the market clearing condition for each good:

$$\gamma_r P_r Y_r = \gamma_u P_u Y_u \quad (3.33)$$

Under monetary policy that keeps $p = \frac{P_u}{P_r} = p_0$, equation (3.33) links Y_r to $Y_u = A_u (\mu^u N_u)^{1-\alpha}$ and therefore to μ^u using the equilibrium value of urban labour supply from equation (3.26).

3.3.3.1 Non-homothetic preferences and changing urban expenditure shares on rural goods

From equation (3.33), one potential way to diminish the link between Y_r and μ^u is to allow γ_u to vary with urban per-capita income. As equation (3.33) indicates, with p set at p_0 , total rural employment N^r would generally increase with μ^u when $\alpha > 0$. Per-capita urban income, from equations (3.12) and (3.26), is decreasing as μ^u increases:

$$\frac{Y_u}{\mu^u} = (1 - \alpha)^{\frac{1-\alpha}{\alpha+\phi}} (A_u)^{\frac{1+\phi}{\phi+\alpha}} (p_0)^{\frac{\gamma^u(1-\alpha)}{\alpha+\phi}} (\mu^u)^{\frac{-\alpha(1+\phi)}{\phi+\alpha}} \quad (3.34)$$

From equation (3.21), urban consumption of rural goods depends on urban per-capita income (at the fixed relative price p_0) and the preference parameter γ_u . The literature on structural transformation (e.g. Herrendorf et al. [2014]) often employs non-homothetic preferences, such as the Stone-Geary utility function, that lead to a declining expenditure share on rural (agricultural) goods as the urban (manufacturing) sector expands and urban per-capita income rises.

In our setting, this can be modeled by allowing γ_u to depend negatively on urban per-capita income. However, that would imply that the fall in per-capita urban income on account of rural-urban migration would actually *increase* urban consumption of rural goods, which would only reinforce the positive relationship between N^r and μ^u .

3.3.3.2 Congestion externalities and urban disamenities

One can thereby conclude, from the goods market clearing condition in equation (3.33), that the only way for the first migration wedge component to favour *less* rural-urban migration being chosen by the planner is if Y_u is decreasing in μ^u due to, e.g. negative congestion externalities introduced by a larger urban population. This is often manifest in higher commuting costs in the urban region due to traffic congestion (see e.g. Brueckner and Lall [2015]).

Suppose agents in the urban region face disamenity costs due to urban congestion that lowers their utility. Spatial models with labour mobility, such as the Rosen-Roback framework discussed in e.g. Glaeser [2008], allow utility-providing amenities to vary across locations. I now allow the utility from urban amenities to depend on urban population size (as in Fajgelbaum and Gaubert [2020]). The utility function for urban residents is now⁹:

$$U^i(C_j^i, N_{ij}; \mu^u) = \ln \left(C_j^i - \frac{N_{ij}^{1+\phi}}{1+\phi} \vartheta(\mu^u) \right) \quad (3.35)$$

where $\vartheta'(\mu^u) > 0$. Hence, an increase in μ^u lowers utility due to the disamenity effect of greater congestion.

⁹I have modeled it this way to obtain a negative effect of higher urban population on output. The key point is that higher urban population should reduce urban residents' utility *and* lower urban output.

The urban labour supply choice is obtained from:

$$N_u^{\phi+\alpha} = \frac{(1-\alpha)P_u A_u (\mu^u)^{-\alpha}}{P^u \vartheta(\mu^u)} \quad (3.36)$$

Then, urban output Y_u is:

$$Y_u = \left[(1-\alpha)^{\frac{1-\alpha}{\alpha+\phi}} (A_u)^{\frac{1+\phi}{\phi+\alpha}} (p_0)^{\frac{\gamma^u(1-\alpha)}{\alpha+\phi}} \right] (\mu^u)^{\frac{\phi(1-\alpha)}{\phi+\alpha}} \vartheta(\mu^u)^{\frac{-(1-\alpha)}{\alpha+\phi}} \quad (3.37)$$

From equation (3.33), we have:

$$\frac{Y_r}{\mu^r} = \frac{\gamma_u}{\gamma_r} \frac{\left[(1-\alpha)^{\frac{1-\alpha}{\alpha+\phi}} (A_u)^{\frac{1+\phi}{\phi+\alpha}} (p_0)^{\frac{\gamma^u(1-\alpha)+\alpha+\phi}{\alpha+\phi}} \right] (\mu^u)^{\frac{\phi(1-\alpha)}{\phi+\alpha}} \vartheta(\mu^u)^{\frac{-(1-\alpha)}{\alpha+\phi}}}{1 - \mu^u} \quad (3.38)$$

If we assume that $\vartheta(\mu^u) = (\mu^u)^\omega$ where $\omega > 0$; then rural per-capita income is decreasing in μ^u if:

$$\omega > \phi + \frac{\phi + \alpha}{1 - \alpha} \frac{\mu^u}{\mu^r}$$

Intuitively, if the disamenity effect of urban congestion is larger, the resulting lower urban labour supply outweighs the positive effect on output due to an increase in urban population.

3.4 Rural output multipliers

In the environment described above, with relative price kept unchanged by monetary policy, urban output affected by urban population size and rural-urban demand linkages, one can consider the impact on rural output of policies that promote migration or transfer resources directly to rural residents, keeping the urban and rural populations fixed¹⁰.

This is salient as many developing countries generally respond to regional shocks by using the following two types of instruments, as opposed to actively promoting migration out of distressed regions. *First*, transfers (cash or in-kind) serve as an inter-regional risk-sharing device (see e.g. IMF [2017] for a discussion in the Indian context).

¹⁰The transfer may thus be interpreted as being conditional on the recipient remaining in the rural region.

Second, rural 'workfare programs' such as employment guarantees provide employment following adverse rural shocks. For instance, the Indian National Rural Employment Guarantee Act (MNREGA) is the largest workfare program in the world and entitles every rural household the right to one hundred days of minimum wage-employment per year. Rural workers predominantly avail of this program during the agricultural lean season (see e.g. [Sukhtankar \[2016\]](#) for a review).

In this section, I will first derive the rural output multiplier associated with migration from the rural to the urban region. I then consider how the multiplier changes when one adds differing expenditure patterns, heterogeneous worker productivity and input-output linkages to the baseline model. I will then derive the rural output multiplier associated with transfers to rural residents under the assumption that labour is restricted to be immobile.

3.4.1 The migration rural output multiplier

One can derive the rural output multiplier, i.e. the variation of Y_r with urban population μ^u , using equations (3.33) – (3.34):

$$Y_r = \frac{\gamma_u}{\gamma_r} \left[(1 - \alpha)^{\frac{1-\alpha}{\alpha+\phi}} (A_u)^{\frac{1+\phi}{\phi+\alpha}} (p_0)^{\frac{\gamma^u(1-\alpha)}{\alpha+\phi}} \right] (\mu^u)^{\frac{\phi(1-\alpha)}{\phi+\alpha}}$$

Thus, the migration rural output multiplier is:

$$\begin{aligned} \frac{\partial Y_r}{\partial \mu^u} &= \frac{\phi(1-\alpha)}{\phi+\alpha} \frac{\gamma_u}{\gamma_r} \left[(1 - \alpha)^{\frac{1-\alpha}{\alpha+\phi}} (A_u)^{\frac{1+\phi}{\phi+\alpha}} (p_0)^{\frac{\gamma^u(1-\alpha)}{\alpha+\phi}} \right] (\mu^u)^{\frac{\phi(1-\alpha)}{\phi+\alpha}-1} \\ \implies \frac{\partial Y_r}{\partial \mu^u} &= \frac{\phi(1-\alpha)}{\phi+\alpha} \frac{Y_r}{\mu^u} \end{aligned} \quad (3.39)$$

The *elasticity* of rural output with respect to urban population size is then:

$$\epsilon_{\mu^u}^{Y_r} = \frac{\phi(1-\alpha)}{\phi+\alpha} < 1$$

This multiplier arises because of the demand linkages between rural and urban sectors that are reflected in equation (3.33). Clearly, in the absence of these linkages ($\gamma_u = 0$), a migration based policy would have *no* impact on rural output, although it

would raise per-capita rural output as μ^r falls. Also, with a linear production function ($\alpha = 0$), the elasticity is unity.

3.4.1.1 Differentiating migrants' propensity to spend on rural goods

Thus far, I have assumed that migrants to the urban region are indistinguishable from prior urban residents. Specifically, their expenditure on rural goods is γ_u , which is identical to prior urban residents. In practice, much rural-urban migration following negative rainfall shocks is temporary in nature. Further, if migrants remit their earnings to their families in the rural region, the propensity to spend on rural goods out of the higher labour earnings is instead $1 - \gamma_r$. I now derive the migration rural output multiplier under this assumption.

Let the urban population μ^u be expressed as the sum of incumbent (μ_i^u) and migrant (μ_m^u) populations, i.e.

$$\mu^u = \mu_i^u + \mu_m^u$$

Denote the relative share of migrants by $m_u = \frac{\mu_m^u}{\mu^u}$.

As before, regional profits are fully taxed and rebated to the corresponding region's residents, hence equation (3.20) continues to hold for all urban residents, migrants and incumbents.

The goods market clearing conditions become:

$$Y_r = \mu^r C_r^r + \mu_i^u C_r^{ui} + \mu_m^u C_r^{um} \quad (3.40)$$

Now, using equation (3.21):

$$C_r^{ui} = \frac{\gamma_u P_u Y_u}{\mu^u P_r}$$

and

$$C_r^{um} = \frac{(1 - \gamma_r) P_u Y_u}{\mu^u P_r}$$

Substituting these into equation (3.40), the goods market clearing condition becomes:

$$\gamma_r P_r Y_r = \left[(1 - m_u) \gamma_u + m_u (1 - \gamma_r) \right] P_u Y_u \quad (3.41)$$

Then, following the same steps used to derive equation (3.39), the migration rural

output multiplier becomes:

$$\begin{aligned} \frac{\partial Y_r}{\partial \mu_m^u} &= \frac{\phi(1-\alpha)}{\phi+\alpha} \frac{Y_r}{\mu^u} + p_0 \frac{(1-\gamma_r-\gamma_u)Y_u}{\mu^u} \frac{\mu_i^u}{\mu^u} \\ \Rightarrow \frac{\partial Y_r}{\partial \mu_m^u} &= \frac{\phi(1-\alpha)}{\phi+\alpha} \frac{Y_r}{\mu^u} + \underbrace{\frac{\mu_i^u(1-\gamma_r-\gamma_u)}{\mu_m^u(1-\gamma_r-\gamma_u) + \mu^u\gamma_u}}_{\text{Additional output effect}} \frac{Y_r}{\mu^u} \end{aligned} \quad (3.42)$$

As equation (3.42) shows, there is an additional effect of migration on rural output when migrants retain their (rural) expenditure shares: an additional migrant increases the migrant share in the urban region, hence there is an extra effect of this higher propensity to spend on rural goods. The difference in rural migrants' versus urban residents' propensities to spend on rural goods accounts for the presence of the $(1-\gamma_r-\gamma_u)$ coefficient in the second term on the RHS of equation (3.42).

3.4.1.2 Heterogeneous worker efficiencies

As the introduction noted, a large part of the argument for migration policies is based on reaping efficiency gains ensuing from allowing heterogeneously skilled workers to sort geographically based on their ability levels in various locations. The model used here has hitherto abstracted from this by assuming that differences in agent type j stem from differing disutility costs from migration. I now introduce differences in latent worker ability by location in a simple fashion and consider how it affects the multiplier.

I follow the treatment in, e.g. [Lagakos et al. \[2018\]](#), [Fajgelbaum and Gaubert \[2020\]](#), that introduces location specific worker heterogeneity based on the Roy model ([Roy \[1951\]](#)). I assume that agents differ in terms of their latent abilities, measured as their efficiency units of labour, in the two regions. For simplicity, I assume that effective labour supplied is identical to actual labour supplied in the rural region. In contrast, agents differ in their productivity in the urban region: the effective labour supplied by a type j worker is $z_j N_{uj}$. Urban productivity z_j is sorted by agent type j in ascending order, hence types corresponding to higher values of j are more productive in the urban region.

Let W_u denote wages per efficiency unit of labour. Unlike in the earlier analysis, differences in worker productivity would lead to variation in labour supply by type.

The labour supply choice of a worker of type j in the urban region solves:

$$N_{uj}^\phi = \frac{z_j W_u}{P_u}$$

From the urban firm's labour demand condition, we can substitute for W_u above:

$$N_{uj} = \left[\frac{(1 - \alpha) z_j P_u A_u (N^u)^{-\alpha}}{P_u} \right]^{\frac{1}{\phi}} \quad (3.43)$$

The urban labour market clearing condition is:

$$N^u = \sum_j \mu_{uj} z_j N_{uj} \quad (3.44)$$

Combining equations (3.43) – (3.44), we can solve for N^u :

$$N^u = \left[(1 - \alpha) p_0^{\gamma_u} A_u \right]^{\frac{1}{\phi + \alpha}} \left(\sum_j \mu_{uj} z_j^{\frac{1 + \phi}{\phi}} \right)^{\frac{\phi}{\phi + \alpha}} \quad (3.45)$$

This implies that urban output is:

$$Y_u = A_u^{\frac{1 + \phi}{\phi + \alpha}} \left[(1 - \alpha) p_0^{\gamma_u} \right]^{\frac{1 - \alpha}{\phi + \alpha}} \left(\sum_j \mu_{uj} z_j^{\frac{1 + \phi}{\phi}} \right)^{\frac{\phi(1 - \alpha)}{\phi + \alpha}} \quad (3.46)$$

As in equation (3.33), the goods market clearing condition implies that rural output is:

$$Y_r = \frac{\gamma_u}{\gamma_r} A_u^{\frac{1 + \phi}{\phi + \alpha}} p_0^{\frac{\phi + \alpha + \gamma_u(1 - \alpha)}{\phi + \alpha}} (1 - \alpha)^{\frac{1 - \alpha}{\phi + \alpha}} \left(\sum_j \mu_{uj} z_j^{\frac{1 + \phi}{\phi}} \right)^{\frac{\phi(1 - \alpha)}{\phi + \alpha}} \quad (3.47)$$

The rural output multiplier from increasing the share of type j agents in the urban region is then:

$$\frac{\partial Y_r}{\partial \mu_{uj}} = \frac{\phi(1 - \alpha)}{\phi + \alpha} Y_r \frac{z_j^{\frac{1 + \phi}{\phi}}}{\left(\sum_j \mu_{uj} z_j^{\frac{1 + \phi}{\phi}} \right)} \quad (3.48)$$

The rural output multiplier from increasing μ_{uj} is then increasing in j as higher agent types are more productive in the urban region. If $z_j = 1 \forall j$, the above expression collapses to equation (3.39).

Intuitively, labour *sorting*, interpreted as the rural-urban migration of those agents who are relatively more productive in the urban region, leads to larger increases in urban output and, through demand linkages, a larger rural output effect. Hence, in this environment, sorting also has beneficial effects on rural output through demand linkages in addition to the usual efficiency gains.

3.4.1.3 Input-output linkages

I have hitherto only considered consumer demand linkages between regions. One might expect that if regional products also served as inputs in the production process, then the migration rural output multiplier would be strengthened. I now confirm this intuition in a simple extension of the baseline model above, following for e.g. the treatment in Guerrieri et al. [2020] and especially the baseline model in the survey by Carvalho and Tahbaz-Salehi [2019].

Denote the rural (urban) input demanded by the urban (rural) good producer by X_u (X_r). The urban production function is modified to:

$$Y_u = A_u (N^u)^{1-\alpha} X_u^\alpha \quad (3.49)$$

with an analogous expression for the rural production function. I derive factor demands for urban producers below, with similar derivations for rural factor demands.

As the production function is of the Cobb-Douglas form, the factor demands are a constant share of revenue. In particular:

$$X_u = \alpha p Y_u$$

Using labour supply choice and rural input demands, this becomes:

$$N_u^\phi = p^{\gamma_u + \frac{\alpha}{(1-\alpha)}} A_u^{\frac{1}{1-\alpha}} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} \quad (3.50)$$

Hence, urban labour supply does not depend on urban population size, unlike in equation (3.26). This also implies that urban aggregate output is a linear function of μ^u .

The other differences with the baseline model are the goods market clearing conditions for the rural and urban goods. For the rural good, this is now:

$$Y_r = C_r^r + C_r^u + X_u$$

As regional firms do not earn profits that are in turn rebated to that region's residents, the consumption shares of rural goods by region k 's residents is now:

$$C_r^k = \gamma_r^k \frac{W_r N_r}{P_r}$$

The goods market clearing condition is then:

$$\left[\alpha + (1 - \alpha) \gamma_r \right] P_r Y_r = \left[\alpha + (1 - \alpha) \gamma_u \right] P_u Y_u \quad (3.51)$$

Then, the migration rural output multiplier is:

$$\frac{\partial Y_r}{\partial \mu^u} = \frac{Y_r}{\mu^u} \quad (3.52)$$

and the elasticity of rural output with respect to urban population size is 1. This is larger than the elasticity in the baseline model, $\frac{\phi(1-\alpha)}{\phi+\alpha} < 1$. This is because rural-urban migration increases urban output when the monetary policy response is limited. It therefore increases the demand for rural goods from urban consumers *and* producers. Hence, stronger consumer and factor demand linkages boost the migration rural output multiplier. One can interpret rural-urban migration as a demand shock that plays a role similar to government spending shocks in the production networks model of [Acemoglu et al. \[2016\]](#). Hence, the migration based urban demand shock would propagate upstream to rural suppliers.

3.4.2 Transfer rural output multiplier

Most developing countries in practice do not promote policies that actively encourage rural-urban migration. The policy response to regional shocks such as a bad rainfall season has been to use transfers and workfare programs instead of actively promoting migration out of the distressed region. Although some countries like China and North Korea explicitly restrict internal migration, one could interpret policies that attempt

to clear urban slums or neglect to provide affordable housing or local public goods to poor urban migrants as indirectly discouraging rural-urban migration. I now derive the rural output multiplier associated with a transfer policy.

3.4.2.1 The transfer rural output multiplier

I assume now that labour is not mobile between the two regions following the transfer, hence rural and urban population levels are fixed at μ^r and μ^u respectively¹¹. This could represent a conditional transfer policy that requires the recipient's presence in the rural region, thereby discouraging rural-urban migration by the recipient. An alternative interpretation is that labour mobility is not immediate due to high relocation costs (for instance), so the transfer rural output multiplier captures the shorter term impact on rural output.

Agents in the rural region, which is assumed to be distressed following a negative labour demand shock that leads to involuntary unemployment, are the recipients of transfers t that are financed by lump-sum taxes (l_u) on urban residents satisfying the budget balance condition:

$$t = \frac{\mu^u l_u}{\mu^r} \quad (3.53)$$

Denote the aggregate rural transfer by $t_{agg} = \mu^r t$. The income of rural residents becomes $\frac{P_r Y_r}{\mu^r} + t$, while that of urban residents is now $\frac{P_u Y_u}{\mu^u} - l_u$.

One can similarly derive the goods market clearing condition:

$$Y_r = \mu^r (1 - \gamma_r) \left(\frac{P_r Y_r}{P_r \mu^r} + \frac{t}{P_r} \right) + \mu^u \gamma_u \left(\frac{P_u Y_u}{P_r \mu^u} - \frac{l_u}{P_r} \right)$$

Using equation (3.53) and simplifying, we obtain:

$$\gamma_r P_r Y_r = \gamma_u P_u Y_u + t \mu^r (1 - \gamma_r - \gamma_u) \quad (3.54)$$

Unlike in equation (3.33), the dependence of rural output Y_r on the transfer t in equation (3.54) is also a function of (endogenous) rural prices, P_r . Hence, the net

¹¹I shall relax this assumption in the numerical illustration below.

effect on rural output of transfers is:

$$\frac{\partial Y_r}{\partial t} = \mu^r \frac{(1 - \gamma_r - \gamma_u)}{\gamma_r} \frac{\partial(\frac{t}{P_r})}{\partial t}$$

In general, when $\alpha > 0$, one can show that P_r is increasing with t , thereby attenuating the impact of transfers on rural output. Now, we consider the simpler case when $\alpha = 0$, i.e. the production function is linear. From the labour demand equation with binding rural DNWR, the price of the rural good is constant:

$$P_r = \frac{\bar{W}}{A_r}$$

Then, in this special case, the transfer rural output multiplier is:

$$\frac{\partial Y_r}{\partial t}|_{\alpha=0} = \mu^r \frac{(1 - \gamma_r - \gamma_u)}{\gamma_r} \frac{A_r}{\bar{W}} \quad (3.55)$$

In general, a higher home bias (lower γ_r, γ_u) would lead to a larger rural output multiplier from a transfer. Unlike the migration rural output multiplier in equation (3.39), even if $\gamma_u = 0$, the transfer rural output multiplier is still positive. One can also compare the transfer rural output multiplier derived here to the transfer multiplier in the dynamic models of, e.g. Farhi and Werning [2016], Chodorow-Reich [2019], which is simply $\frac{1-\gamma_r}{\gamma_r}$. In those papers, the region receiving a transfer is a small part of the union, prices are not flexible and the agents are Ricardian, so the γ_u coefficient drops out above.

Migration and restricted transfer policies

Inspecting equations (3.39) and (3.55) helps relate transfer and migration policies¹². This relation is based on the assumption that the transfer policy simultaneously restricts labour mobility in the short-term, hence the policymaker is comparing the stimulative effects of a policy encouraging labour mobility to a policy that restricts labour mobility and instead directly supplements rural incomes.

If the propensity of migrants to spend on rural goods is diminished, then the demand

¹²For the same transfer outlay, the migration rural output multiplier in equation (3.39) is also multiplied by $\frac{\partial \mu^m}{\partial t}$.

Table 3.1: Parameters

<i>Parameter</i>	σ	γ_r	γ_u	α	ϕ	A_r	A_u
<i>Value</i>	2	0.4	0.4	0.2	2	1.5	2

linkages that would boost the migration rural output multiplier are weak and from a stimulative perspective, migration is a less effective response. In contrast, even if $\gamma_u = 0$, a transfer policy would augment rural output although some of the effect would be weakened by an increase in the rural good price. The greater the home bias in *each* region, the larger is the multiplier effect of a transfer policy as opposed to a migration policy.

3.5 Numerical illustration

I now illustrate the magnitude of the migration inefficiency wedge in equation (3.31) and the multipliers derived in section 3.4. I calibrate the productivity and home bias parameters based on the dynamic model in Hnatkovska and Lahiri [2018]. As in that paper, I use a CRRA utility function in the calibration and also assume that there are no wealth effects on labour supply. The parameter values are listed in table 3.1.

3.5.1 The migration wedge

I evaluate the migration wedge in equation (3.31) and associated income subsidy in equation (3.32) that internalizes the aggregate demand externality. In order to do so, I first compute the spatial equilibrium of the model when all prices and wages are flexible: this serves as the benchmark relative price that the monetary authority keeps fixed. I then introduce a negative shock to rural productivity A_r that induces the rural DNWR constraint to bind and compute the spatial equilibrium allocation, migration wedge and corrective income subsidy. In order to avoid corner solutions for urban or rural population shares, I allow for the utility cost to vary with population shares and then compute the regional population distribution that equates regional utilities in a spatial equilibrium.

For instance, when rural labour productivity falls to $A_r = 1.48$ from a baseline value of 1.5, the corrective subsidy is 10.5% of the urban consumption level. The wedge

Table 3.2: Negative rural shocks, corrective subsidy and multipliers

A_r	Subsidy/ C^u	Subsidy without amplification/ C^u	Migration rural output multiplier
1.48	0.105	0.064	2.3
1.46	0.222	0.113	2.29
1.44	0.4	0.19	2.28

and the subsidy are both monotonically decreasing in A_r . Table 3.2 illustrates this pattern for different levels of A_r .

As discussed in section 3.3, the migration wedge is amplified relative to the corresponding expression in Farhi and Werning [2014]. This is because output in rural and urban regions also depends on urban population size. The amplifying factor is sizable and is given by: $\frac{\phi}{\phi+\alpha} \frac{\mu^r}{\mu^u}$. In the numerical illustrations conducted, this factor exceeds unity. The corrective subsidy *excluding* this amplifying term is thus considerably smaller than the results reported above that include the amplifying term, as reported in the third column of table 3.2.

3.5.2 Multipliers

For the parameter values in table 3.1, the elasticity of rural output with respect to urban population size is 0.72. Inspecting equation (3.39), the migration rural output multiplier is decreasing in μ^u and increasing in Y_r . The larger the negative rural productivity shock (lower A_r), the smaller is rural output and the greater is the urban population share. Hence, the migration rural output multiplier is decreasing in A_r , as seen in table 3.2.

In the model with diminishing marginal product of labour ($\alpha > 0$), the transfer rural output multiplier depends on whether labour can move in response to transfer payments. If one assumes, as in section 3.4.2, that labour is immobile in the short-term following transfer payments, then the transfer rural output multiplier when transfers are raised from 10% to 15% is approximately 2.

However, if labour is assumed to be mobile in the short-term under a transfer policy, then rural output actually *decreases* with transfers. From equation (3.54), one notes that now μ^u and μ^r also vary with the transfer t . Then, rural output might decrease with transfers under free mobility because the price of the rural good rises and the urban population size falls in response to rural transfers.

Welfare gains for rural residents from transfers

These are measured as the percentage increase in rural residents' utility from a transfer policy, evaluated under the assumption that labour cannot move immediately following transfers. I evaluate these gains when rural productivity A_r falls to 1.48 from 1.5 and the transfer is 10%. The percentage welfare gain for *rural* residents is approximately 6.4%.

If welfare gains are instead measured in consumption equivalent terms, the percentage increase in consumption for rural residents to be indifferent between a no-transfer versus a 10% transfer policy is 6.77%. The welfare gain for rural residents (under both measures) is increasing in the size of the transfer.

3.6 Conclusion

This paper considers the efficiency and stimulative effects of rural-urban migration following adverse rural labour demand shocks that lead to binding downward rural wage rigidity in an environment with limited monetary policy responsiveness. It uses the analytical framework of [Farhi and Werning \[2014\]](#) to confirm and amplify that paper's result on the general inconsistency of constrained efficient and individual labour mobility decisions. It shows that the plausible finding of inefficiently low migration out of the depressed (rural) region is extremely robust and could be broken by, for instance, strong congestion externalities arising due to migration. It then considers the rural output multiplier from rural-urban migration under differing scenarios. Finally, it relates this multiplier to the multiplier arising from an alternative policy based on transfers to rural residents.

An important objective of the paper is to apply a flexible macroeconomic approach to bear upon the recent discussion on migration policies in the development economics literature. It particularly highlights the argument for migration policies as a stabilizing tool following adverse regional shocks beyond the standard efficiency gains from sorting. An important next step would be to calibrate a dynamic version of the model and evaluate the demand externality and associated corrective migration policy therein.

3.7 Appendix

I first derive the equilibrium allocations when the rural DNWR constraint does not bind and then when it is binding. I then show that labour mobility decisions are constrained inefficient when preferences are separable between consumption and labour.

3.7.1 Equilibrium allocations with and without a binding rural DNWR constraint

3.7.1.1 Rural DNWR constraint does not bind

When the rural DNWR constraint does not bind, the desired and actual rural labour supply choices coincide, $N_r = N_r^s$, and N_r is given by equation (3.7):

$$N_r^{\phi+\alpha} = (1 - \alpha)p^{-\gamma_r} A_r (\mu^r)^{-\alpha}$$

Urban labour supply is given by the analogous expression in equation (3.26):

$$N_u = \left((1 - \alpha)p^{\gamma_u} A_u (\mu^u)^{-\alpha} \right)^{\frac{1}{\alpha+\phi}}$$

Finally, the goods market clearing condition is:

$$\gamma_u p (\mu^u N_u)^{1-\alpha} = \gamma_r (\mu^r N_r)^{1-\alpha}$$

Substituting the expressions for N_u and N_r into the goods market clearing condition, one obtains:

$$p^{1+\frac{(\gamma_u+\gamma_r)(1-\alpha)}{(\phi+\alpha)}} = \frac{\gamma_r}{\gamma_u} \left(\frac{A_r}{A_u} \right)^{\frac{1+\phi}{\phi+\alpha}} \left(\frac{\mu^r}{\mu^u} \right)^{\frac{(1-\alpha)\phi}{(\phi+\alpha)}} \quad (3.56)$$

With labour mobility, equation (3.11) determines regional population shares. With a negative productivity shock in the rural region (e.g. a bad rainfall season), A_r falls and the relative price p also falls from equation (3.56). Hence, a negative rural supply shock raises the price of the rural good and the relative price p falls.

3.7.1.2 Rural DNWR constraint binds

I now consider the equilibrium allocation when the rural DNWR constraint binds. The treatment follows section 3.2.2.1. Hence, rural labour supply is determined by rural labour demand at the rigid nominal wage \bar{W} and is given by equation (3.23):

$$N_r = \left(\frac{(1-\alpha)P_r A_r}{\bar{W}} \right)^{\frac{1}{\alpha}} (\mu^r)^{-1}$$

Substituting urban labour supply from equation (3.26) and the above into the goods market clearing condition, we have:

$$\gamma_u p \left((1-\alpha)p^{\gamma_u} A_u (\mu^u)^{\phi} \right)^{\frac{1-\alpha}{\alpha+\phi}} = \gamma_r \left(\frac{(1-\alpha)P_r A_r}{\bar{W}} \right)^{\frac{(1-\alpha)}{\alpha}} \quad (3.57)$$

Unlike in the case where the rural DNWR constraint does not bind, we note from equation (3.57) that the equilibrium relative price is affected by monetary policy. A suitable choice of P_u can achieve the first-best (flexible rural wage) allocation with a non-binding rural DNWR constraint.

Specifically, P_u chosen so that the relative price satisfies equation (3.56) at the new, lower level of A_r . As p satisfies equation (3.56), N_u is identical to its flexible wage equilibrium value. Then, from the goods market clearing condition, N^r is also equal to its flexible wage equilibrium value.

3.7.2 Separability of preferences between consumption and labour

The analysis throughout has employed non-separable preferences of the GHH kind that rule out wealth effects on labour supply. I now briefly consider how the results are affected when preferences are instead separable.

The utility function now is:

$$U(C_j^i, N_{ij}) = \log(C_j^i) - \frac{N_{ij}^{1+\phi}}{1+\phi}$$

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This amends equation (3.6) to:

$$C_j^i (N_{ij}^s)^\phi = \frac{W_i}{P^i} = \frac{(1 - \alpha) P_i A_i (N^i)^{-\alpha}}{P^i}$$

With the assumption that the utility function represents the same preference ordering for all agents of a given type and that all regional profits are taxed away and rebated to the corresponding region's residents, the above becomes:

$$\frac{P_i Y_i}{P^i \mu^i} (N_i^s)^\phi = \frac{(1 - \alpha) P_i A_i (N^i)^{-\alpha}}{P^i}$$

With $N^i = \mu^i N_i$ when $N_i = N_i^s$, this becomes:

$$(N_i^s)^{1+\phi} = (1 - \alpha)$$

Hence, in the urban region, N_u is independent of the relative price p and urban population μ^u . In the rural region, consider the case where DNWR binds. Rural labour demand satisfies equation (3.23):

$$N^r = \left(\frac{(1 - \alpha) P_r A_r}{\bar{W}} \right)^{\frac{1}{\alpha}}$$

The goods market clearing condition and monetary policy that affects P_u and the relative price together determine P_r in equilibrium:

$$\gamma_u p A_u \left((1 - \alpha)^{\frac{1}{1+\phi}} \mu^u \right)^{1-\alpha} = \gamma_r A_r \left(\frac{(1 - \alpha) P_r A_r}{\bar{W}} \right)^{\frac{1-\alpha}{\alpha}}$$

Regarding the results derived in the paper under GHH preferences, note from the goods market clearing condition that Y_r is still an increasing function of μ^u :

$$Y_r = \frac{\gamma_u}{\gamma_r} p A_u \left((1 - \alpha)^{\frac{1}{1+\phi}} \mu^u \right)^{1-\alpha}$$

Therefore, the first term in the migration wedge component for the rural region will still have N_r as an increasing function of μ^u , as was the case with GHH preferences. Hence, constrained efficient migration decisions are inconsistent with free mobility when preferences are assumed to be separable.

Also, the migration rural output multiplier becomes:

$$\frac{\partial Y_r}{\partial \mu^u} = (1 - \alpha) \frac{Y_r}{\mu^u}$$

Compared to equation (3.39), note that the elasticity of rural output with respect to urban population under separable preferences is:

$$\epsilon_{\mu^u}^{Y_r}|_{\text{Separable}} = (1 - \alpha) > \frac{\phi(1 - \alpha)}{\phi + \alpha} = \epsilon_{\mu^u}^{Y_r}|_{\text{GHH}}$$

With GHH preferences, the migration rural output multiplier is lower as an increase in migration and urban population is offset by a decrease in urban labour supply N_u . With separability of preferences, urban labour supply is independent of μ^u , hence there is no offsetting force on aggregate urban and linked rural output.

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